Math 410, Spring 2026 Professor Rob Benedetto

Homework #1

Due **Friday, January 30** in Gradescope by **11:59 pm ET**

READ Sections 1.1–1.2 in Cox

WATCH 1. Video 1: Ferrari and the Quartic (8:09)

2. Video 2: Review of Polynomial Rings (22:09)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (6 points) Cox, Section 1.1, Problem 3:
Prove that when $p = 0$, Cardano's formulas still correctly give the roots of the cubic equation $y^3 + py + q = 0$.

Problem 2. (10 points) Cox, Section 1.1, Problem 6:
Consider the equation $x^3 + x - 2 = 0$. Note that $x = 1$ is a root.

(a): Use Cardano's formula to prove that $\sqrt[3]{1 + \frac{2}{3}\sqrt{\frac{7}{3}}} + \sqrt[3]{1 - \frac{2}{3}\sqrt{\frac{7}{3}}} = 1$.

(b): Verify that $\left(\frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{7}{3}}\right)^3 = 1 \pm \frac{2}{3}\sqrt{\frac{7}{3}}$, where the two $\pm$'s are either both $+$ or both $-$, and then use this fact to prove the formula in part (a) again.

Problem 3. (6 points) Cox, Section 1.2, Problem 4:
We say that a cubic $x^3 + bx^2 + cx + d$ has a *multiple root* if it can be factored as $(x - r_1)^2(x - r_2)$. Prove that $x^3 + bx^2 + cx + d$ has a multiple root if and only if its discriminant is zero.

Problem 4. (8 points) Cox, Section 1.2, Problem 5:
Define $\sqrt{\Delta} = (x_1 - x_2)(x_1 - x_3)(x_2 - x_3)$. Prove that an even permutation of $\{x_1, x_2, x_3\}$ takes $\sqrt{\Delta}$ to $\sqrt{\Delta}$, while an odd permutation takes $\sqrt{\Delta}$ to $-\sqrt{\Delta}$.

Problem 5. (10 points) Cox, Section 2.1, Problem 1:
Let F be a field. In the ring $F[x, y]$, prove that the ideal

$$\langle x, y \rangle = \{xg + yh \mid g, h \in F[x, y]\}$$

is *not* a principal ideal. (Note: you may assume without proof that it *is* an ideal.)

(Office Hours and optional challenge problems on next page)

Optional Challenges (do NOT hand in): Cox Problems 1.1 #8 and 1.2 #2

Questions? You can ask in:

Class: MWF 9:00am – 9:50am, SCCE C101

My office hours: in my office (SMUD 406):
Mon 2:00–3:30pm
Tue 1:30–3:15pm
Fri 1:00–2:00pm

Also, you may email me any time at `rlbenedetto@amherst.edu`