Math 350, Section 01, Spring 2025: Groups, Rings and Fields

MWF 9:00 – 9:50am, SMUD 014

Webpage: `https://rlbenedetto.people.amherst.edu/math350/`
(Also linked from the Math 350 moodle page.)

Instructor: Rob Benedetto
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Office Hours: Tue 1:30–3:00pm; Wed 1:30–3:00pm; Fri 1:30–2:30pm; or by appointment.

The Math 350 Fellows (or "TAs"): Claire Callon, Aidan Curtis, and Kevin Dai

Claire:	Email: `ccallon25@amherst.edu`	Office hours: TBA
Aidan:	Email: `acurtis25@amherst.edu`	Office hours: TBA
Kevin:	Email: `kdai25@amherst.edu`	Office hours: TBA

Text: D. Saracino, *Abstract Algebra: A First Course*, **second edition**, Waveland Press 2008.
Available via Brytewave on the Math 350 moodle page

Exams:

- **Midterm 1**: **Wednesday, March 5**, in class.
- **Midterm 2**: **Wednesday, April 9**, in class.
- **Final**: **Take-home; details TBA**

The only excuses for missing an exam are incapacitating illness, religious conflict, or the like.

Using calculators, cell phones, etc. during an in-class exam is CHEATING.
Consulting any outside source during any exam is CHEATING.
Copying homework solutions, from any source, is CHEATING.
All cheating will be prosecuted, potentially leading to an F in the course.

Homework:

- Reading from Saracino will be assigned each week, along with short videos to watch.
- Problem sets will be due (usually) twice a week, **on Gradescope by 11:59pm ET**.
  See Homework 0, and see page 3 of this syllabus, for important homework information.

Grading:

- **Effort**: 5%
- **Problem Sets**: 20%
- **Midterm Exams**: Better one: 25%. Worse one: 15%. (Total 40%.)
- **Final Exam**: 35%

"Effort" is a combination of class attendance (including being on time), class participation, and handing in problem sets. It is not computed linearly; a student deficient in any one of those areas will get a very low Effort grade. (See pages 2 and 4 of this handout for more on attendance and participation.)

"Problem Sets" means actual grades on the problem sets. Late problem sets will be marked down substantially in the Problem Sets portion of your grade (see the webpage for details); but all problem sets handed in by the last day of classes count towards Effort.

If one component of an individual student's course grade is substantially higher or lower than their other grades, and if the student's Effort grade is strong, I will tweak the above percentages a little for that student to favor the better grades. Overall course grades will be curved.

About Cell Phones and Mobile Devices

Cell phones and similar devices have no place in my classroom. Don't use them. Not for talking, not for texting, not for anything. So at every class:

Silence your cell phone, put it away, and pay attention.

Necessary Background

The prerequisites for this course are multivariable calculus (Math 211) and linear algebra (Math 271 or Math 272). While we will occasionally use a little linear algebra material (like matrices), the more important requisite is exposure to both abstract concepts and writing proofs. I don't expect you to feel completely fluent with proofs yet, but if you struggled with proofs in linear — for example, if you got a B+ or lower, or even an A- but the proofs part didn't go so well — you might be better served by taking a course like Math 220 (Reasoning and Proof) before you try Math 350 or 355, or perhaps another proof-based 200-level Math elective.

Course Content

Abstract algebra is the study of sets that have "operations" on them, and the "algebraic" rules that they obey. In linear algebra, you saw vector spaces, which are sets having an addition operation and a scalar multiplication operation satisfying certain axioms. Math 350 focuses mainly on groups, which are sets having one operation satisfying a certain (short) list of axioms. We will also encounter rings and fields, which are groups that have a second operation along with more axioms.

The idea here is **not** to write down some random axioms and see what follows. Instead, abstract algebra grew out of the observation that the same patterns and ideas arose in vastly different realms of the mathematical world. For example, the set of integers, the set of rational numbers, any vector space, the set of ways of rotating a molecule, certain sets of matrices, the set of possible shuffles of a deck of cards, sets of functions, and many other sets, can all be viewed as groups. So if we can understand groups in general, we can understand many other specific objects as well.

Here's a more detailed list of some of what we'll see.

- Sections 0–1 are review of basic concepts about **sets, functions, and the integers**, with a few new ideas here and there. We'll mostly skip these sections but refer to them as the need arises.
- In Sections 2–5, where we'll start, we'll define **groups** and discuss their basic properties and features, including **subgroups**. We'll see lots of examples along the way.
- In Sections 6–9, we'll introduce **symmetric groups**, a fundamental family of examples. We'll also see more complicated structures built using groups, namely the relatively tame notion of **direct products** of groups and the often-confusing notion of **cosets** of a subgroup.
- In Section 10, we'll learn to **count** things. This is harder than it sounds. Fortunately, it's also more interesting than it sounds.
- In Sections 11–13, we'll learn about **normal subgroups**, **quotient groups**, and **homomorphisms**, which are all-important and intimately related objects.
- We'll skip past Sections 14 and 15, which cover deeper theorems on the structure of finite groups: The structure theorem of finite abelian groups, and the Sylow ("SEE-loh") Theorems.
- In Sections 16–18, we'll introduce **rings** and **fields**, along with their **homomorphisms**, **ideals** (roughly analogous to normal subgroups), and **quotient rings** (analogous to quotient groups).
- Finally, in Sections 19–21, we'll study rings of **polynomials** and related topics, including **finite fields** and **unique factorization**.

Class Participation and Classroom Dynamics

Class participation is part of the Effort portion of your grade. If you are quiet by nature, don't worry; as long as you attend class devotedly, pay close attention, and do the homework, you will get full Effort credit. That said, finding your voice in class helps you learn better. So for your own benefit, **speak up, ask questions, and even try to answer questions I may ask in class**.

On the flip side, **respect your fellow students** both in and out of the classroom. Treat every person's ideas the same way you would if it had been me or the college president sharing them.

Homework

Your homework consists of **ALL THREE** of reading the book and handouts, watching the assigned videos, **AND** doing problem sets. Skim textbook sections **before** the corresponding lecture, and then go back and read more carefully to solidify your understanding and to help with the problem sets.

Please note the following **Important Problem Set Rules:**

1. Problem sets are due **on Gradescope**, each by its specified deadline.
2. You must "tag" your problems in Gradescope, marking where each problem's solution appears.
3. **Write legibly**, and organize your work clearly. **Make it a pleasure to read!**
4. If you worked with other students or got help from a source other than me, the TAs (Aidan, Claire, and Kevin), the book, or other course resources, then say so explicitly on the first page of your problem set. (See the discussion below on the Statement of Intellectual Responsibility.)
5. The Problem Sets grade for any late assignment will be substantially reduced. The later it is, the greater the reduction; see the course webpage under "Problem Set Rules" for details.

See Homework 0 and the handout on Gradescope Instructions for more information on Gradescope.

I am often willing to grant penalty-free extensions on problem sets; but see "Attendance, Extensions, and Extra Office Hours" on page 4.

I strongly encourage you to work on problem sets together, in pairs or small groups, provided you follow the common-sense guidelines below.

About the Statement of Intellectual Responsibilty

Exams: Your work must be entirely your own, so no looking at other people's papers, no communication, and no outside help. For the in-class midterms, no books, notes, calculators, or other resources are allowed, either. For the take-home exam, you may use only course materials, **your own** notes, and the textbook; you may also consult me (Prof. R. Benedetto), but not the TAs or anyone else. No other books, notes, online resources, or communications with other people are allowed.

Problem sets: I urge you to collaborate with each other, under the following ground rules:

1. If you collaborate with, say, Jane and Joe, write a note on the front of your problem set saying, "I worked with Jane and Joe." Use similar notation if you got help from a tutor, fellow student, another professor, another book, the web, etc. However, you do **not** need to write about help you got from me, the Math 350 TAs (Aidan, Claire, and Kevin), the textbook, or course materials.
2. Working together does not mean that Joe does the first half of the problem set and Jane does the second half; everyone should work on every problem.
3. Each student must hand in their own problem set; you can't hand in a single packet as the work of multiple people. For Homework 0, by the way, "kernel" is one of the secret words.
4. Each student must write up each problem **in their own words**. Working together means discussing the problems. Copying someone else's solution (even when the source doesn't mind) is plagiarism and a violation of intellectual responsibility.

A common question: What if Joe asks Jane about a homework problem she has already solved? If Joe copies Jane's solution, both Joe and Jane would be guilty of academic dishonesty, leading to an F on the assignment for both of them, and possibly in the whole course. Instead, Jane can explain her solution to Joe (even showing him what she wrote), before Joe writes up his own solution himself, **in his own words**. Joe would then have to write that he got help from Jane (see rule 1 above), but Jane doesn't need to write anything unless she also got help in return.

If at any time you aren't sure about what's OK and what's not as far as intellectual responsibility is concerned for this course, talk to me about it.

Class Attendance, Extensions, and Extra Office Hours

Attendance: You should be at every class meeting, and you should be on time. Of course, if you're sick, are in mandated isolation, have a religious conflict, or the like, just let me know (in advance, when possible). One or two accidental misses are OK, too; oversleeping can happen, but it should be **rare**. Otherwise, however,

I expect you to be in class, and on time, for every class meeting.

I don't plan to take formal attendance, but I will easily be able to tell who misses class too much; those students' Effort grades will take a hit. (And after more than just a very few unexcused absences, or showing up late too much, your Effort grade will truly plummet, since I do not compute it linearly.)

Extensions: You may request **two** homework extensions over the course of the semester, for up to 48 hours. To claim an extension, you must:

1. Request the extension (by email, or in person) **no later than 8pm ET on the due date**,
2. Not have used both your extensions yet, and
3. Have been attending class devotedly and meeting homework deadlines.

Note: you do **not** need to provide an excuse or reason for your extension request.

Office Hours: You are always welcome to attend my regularly scheduled office hours. In addition, **IF you have been attending class and doing the homework**, you are also welcome to make appointments to see me **outside** of my regularly scheduled office hours.

Getting Help

If you're ever stuck or confused, seek help immediately:

Office Hours: Stop by (unannounced) to see me during my scheduled office hours, or make an appointment to see me another time.

Math 350 TAs: Visit the TAs' office hours, too. See the course website for details.

The QCenter: **Allison Tanguay** of the the Moss Quantitative Center also offers Math 350 help, both for unscheduled drop-ins and for scheduled appointments. See the course website for details of the times and locations of her drop-in hours, and also the link there (on "QCenter Help") to schedule an appointment with her.

Tutoring: If you feel you need regular one-on-one help from a peer tutor, we can probably set that up. To do so, talk to me first.

Advice, and What to Expect

For many of you, Math 350 will be the most abstract and proof-intensive math course you've taken. Courses like Math 271 or 272 or 220 are excellent preparation, but still, start working on each problem set **the same day it is assigned**; do *not* put it off until the due date. The assignments will be long and challenging, and you may want to seek help (e.g., office hours or asking questions in class). So every week, **read and think about ALL of the problems several days in advance**.

The written homework will consist almost entirely of proofs and abstract problems, requiring more thinking time (as opposed to calculation time) than you may be used to. Your solutions to such problems will require **words, complete sentences, and well-written paragraphs**, but usually very little calculation. Get in the habit of working things out on scratch paper in advance, figuring out what needs to be said, what **doesn't** need to be said, and what **order** everything should come in. You don't need to explain the intuition; you need to **prove the theorem**.

And if you're ever feeling lost, there's always office hours: mine, the Math 350 Fellows', and Allison Tanguay's.

On the other hand, abstraction and proof do not always mean difficult or bizarre. In fact, abstract algebra is often quite elegant and beautiful. My hope is that all of you will find the subject of abstract algebra to be as much fun as I do.