Optional Handout: Definition Practice

You've already heard me recommend that you write down definitions and theorems in Math 350, both to have them all in one place for reference later, and to help internalize and understand them. (That is, keeping a definitions-and-theorems journal.)

This optional handout is a sample of what I mean; if you like it, try doing the same yourself with future new concepts. This page has blanks to fill in; page 2 shows them filled in. Try filling it all in without looking at page 2, but then come back and write it out on a separate sheet of paper while you **are** looking at page 2, so that you yourself have written out all these definitions correctly in one place. Use language/notation like $\exists$, $\forall$, s.t., etc. whenever appropriate.

Definition Let G be a group, let $x \in G$, and let $n \in \mathbb{Z}$. Then x^n denotes

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Definition: Let G be a group, and let $x \in G$. Then $\langle x \rangle$ denotes the set

Practice: Write down formally what it means to say $a \in \langle x \rangle$.

Definition: Let G be a group. Then we say G is cyclic if

Practice: If we're given a group G and some $x \in G$, write down a skeleton of a proof that x is a generator for G.

Definition: Let $m, n \in \mathbb{Z}$ with $n \geq 1$. We say $m|n$ if

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Definition: Let $j, k, n \in \mathbb{Z}$ with $n \geq 1$. We say $j \equiv k \pmod{n}$ if

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Practice: If we're given a group G and a subset $H \subseteq G$, write down a skeleton of a proof that H is a subgroup of G.

Filled-in Definitions

Definition Let G be a group, let $x \in G$, and let $n \in \mathbb{Z}$. Then x^n denotes

$$\boxed{x^n = \begin{cases} \underbrace{xx\cdots x}_{n} & \text{if } n \geq 1, \\ e & \text{if } n = 0, \\ \underbrace{x^{-1}x^{-1}\cdots x^{-1}}_{-n} & \text{if } n \leq -1. \end{cases}}$$

Definition: Let G be a group, and let $x \in G$. Then $\langle x \rangle$ denotes the set

$$\boxed{\langle x \rangle = \{x^n \,|\, n \in \mathbb{Z}\}}$$

Practice: Write down formally what it means to say $a \in \langle x \rangle$.

$$\boxed{a \in \langle x \rangle \text{ means } \exists n \in \mathbb{Z} \text{ s.t. } a = x^n}$$

Definition: Let G be a group. Then we say G is cyclic if

$$\boxed{\exists x \in G \text{ s.t. } G = \langle x \rangle}$$

or alternatively, if you prefer, equivalently we can say:

$$\boxed{\exists x \in G \text{ s.t. } \forall a \in G, \exists n \in \mathbb{Z} \text{ s.t. } a = x^n}$$

Practice: If we're given a group G and some $x \in G$, write down a skeleton of a proof that x is a generator for G.

Proof. Given $g \in G$,
$\ldots\ldots$
Let $n = \boxed{} \in \mathbb{Z}$.
Then $g = \cdots\cdots = x^n$ QED

Definition: Let $m, n \in \mathbb{Z}$ with $n \geq 1$. We say $m|n$ if

$$\boxed{\exists q \in \mathbb{Z} \text{ s.t. } n = qm}$$

Definition: Let $j, k, n \in \mathbb{Z}$ with $n \geq 1$. We say $j \equiv k \pmod{n}$ if

$$\boxed{n|(j-k)}$$

or alternatively, if you prefer, equivalently we can say:

$$\boxed{\exists q \in \mathbb{Z} \text{ s.t. } j - k = qn}$$

Practice: If we're given a group G and a subset $H \subseteq G$, write down a skeleton of a proof that H is a subgroup of G.

Proof. **(Nonempty)** Observe that $\boxed{} \in H$, because .... So $H \neq \varnothing$.
(Closure) Given $h_1, h_2 \in H$. Then $h_1 h_2 = \cdots\cdots = \boxed{} \in H$
(Inverses) Given $h \in H$. Then $h^{-1} = \cdots\cdots = \boxed{} \in H$ QED