

Homework #6Due **Tuesday, February 18** in Gradescope by **11:59 pm ET****READ** Sections 5–6 in Saracino and the **Proof of the Subgroup Theorem** handout

- WATCH** 1. Required: Video 13: Subgroups of Cyclic Groups (15:12)
2. Optional: Video 14: Combining Infinitely Many Groups (20:32)

WRITE AND SUBMIT solutions to the following problems.**Problem 1.** (24 points) Saracino, Section 5, Problem 5.5(a): Find all the subgroups of Q_8 .

[**Note 1** from RLB: of course, also prove that your list is complete. Hint: it should turn out that there are six subgroups: one each of orders 1, 2, and 8, and three of order 4.]

[**Note 2** from RLB: the (unassigned) part (b) in the book asks you to prove that Q_8 is a nonabelian group, but all of its proper subgroups are cyclic. This is just a matter of noting that each of the subgroups you listed, besides Q_8 itself, is cyclic; but still, it's an interesting observation.]

Problem 2. (8 points) Saracino, Section 5, Problem 5.7:

Let $G = \langle x \rangle$ be a cyclic group of finite order n . Show that for any integer $m \in \mathbb{Z}$, the element x^m is a generator of G if and only if $(m, n) = 1$.

[Saracino has more commentary, and you should read that, but that commentary is not part of what you need to do to complete this problem.]

Problem 3. (5 points) Saracino, Section 5, Problem 5.8:

Let $G = \langle x \rangle$ be a cyclic group of order 144. How many elements are there in the subgroup $\langle x^{26} \rangle$?

Problem 4. (10 points) Saracino, Section 5, Problem 5.11:

Let G be an abelian group, and let $n \geq 1$ be a positive integer. Let $H = \{x \in G \mid x^n = e\}$.

[Or as Saracino phrases it, let H be the subset of G consisting of all $x \in G$ such that $x^n = e$.]

Prove that H is a subgroup of G .

Problem 5. (8 points) Saracino, Section 5, Problem 5.14:

Let H, K be subgroups of a group G . Prove that $H \cap K$ is also a subgroup of G .

Problem 6. (10 points) Saracino, Section 5, Problem 5.23:

Let G be a group, and let $g \in G$. Define the **centralizer** of g in G to be the subset

$$Z(g) = \{x \in G \mid xg = gx\}.$$

Prove that $Z(g)$ is a subgroup of G .

(Optional Challenges and Office Hour Information on next page)

Optional Challenges (do NOT hand in): Saracino Problems 5.26, 5.28

Questions? You can ask in:

Class:

Section 01: MWF 9:00–9:50am, SMUD 014

Section 02: MWF 11:00–11:50am, SMUD 205

My office hours: in my office (SMUD 406):

Tue 1:30–3:00pm

Wed 1:30–3:00pm

Fri 1:30–2:30pm

Allison Tanguay's QCenter Drop-in Hours, in SMUD 208:

MWF 10am – noon

TuTh 1pm – 4pm

Math Fellow Drop-in Hours, in SMUD 208:

Sun 6:00–7:30pm (Kevin)

Mon 7:30–9:00pm (Claire)

Tue 8:30–10:00pm (Aidan)

Wed 7:30–9:00pm (Claire)

Thu 8:30–10:00pm (Aidan)

Fri 6:00–7:30pm (Kevin)

Also, you may email me any time at rlbenedetto@amherst.edu