

**Homework #4**Due **Tuesday, February 11** in Gradescope by **11:59 pm ET****READ** Section 4 in Saracino**WATCH 1.** Required: Video 8: The Euclidean Algorithm (13:31)2. Optional: Video 9: Proof of the Euclidean Algorithm (10:38)**WRITE AND SUBMIT** solutions to the following problems.**Problem 1.** (8 points) Saracino, Section 4, Problem 4.5:Let  $G$  be a group, and let  $x \in G$  be an element of order 18. Find the orders of  $x^2, x^3, x^4, x^5, x^{12}$ .**Problem 2.** (10 points) Saracino, Section 4, Problem 4.6:List all the elements of  $(C_{45}, \oplus)$  that are of order 15. As always, justify your answer.[Note from RLB: recall I write  $C_{45}$  for what Saracino calls  $\mathbb{Z}_{45}$ .]**Problem 3.** (15 points) Saracino, Section 4, Problem 4.10(a):Let  $G = \{1, 2, 3, 4, 5, 6\}$  and define an operation  $\odot$  on  $G$  by  $a \odot b = \overline{ab}$ , the remainder of  $ab$  (mod 7). For instance,  $2 \odot 4 = \overline{8} = 1$ , and  $5 \odot 6 = \overline{30} = 2$ . Prove that  $(G, \odot)$  is a group.[*Suggestion:* for associativity, use the result of HW 3, Problem 6.]**Problem 4.** (10 points) Saracino, Section 4, Problem 4.10(b):Is the group  $(G, \odot)$  above cyclic? Prove or disprove.**Problem 5.** (12 points) Saracino, Section 4, Problem 4.20:Let  $G$  be a group and  $a \in G$ . We say an element  $b \in G$  is a *conjugate* of  $a$  if there exists  $x \in G$  such that  $b = xax^{-1}$ . Show that any conjugate of  $a$  has the same order as  $a$ .*Suggestions* for problem 5: For one thing: use HW 3, Problem 7.For another thing: to show  $o(g) = o(h)$ , start by defining two sets  $S = \{n \geq 1 \mid g^n = e\}$  and  $T = \{n \geq 1 \mid h^n = e\}$ , and then prove that the sets  $S$  and  $T$  are equal. Then  $o(g)$  is the smallest element of  $S$ , or  $\infty$  if  $S = \emptyset$ , and similarly for  $o(h)$  for  $T$ .**Problem 6.** (12 points) Saracino, Section 4, Problem 4.21: Let  $G$  be a group and  $x, y \in G$ . Prove that  $o(xy) = o(yx)$ .

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**Optional Challenges (do NOT hand in):** Saracino Problems 4.22, 4.23

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**Questions?** You can ask in:

**Class:**

Section 01: MWF 9:00–9:50am, SMUD 014

Section 02: MWF 11:00–11:50am, SMUD 205

**My office hours:** in my office (SMUD 406):

Tue 1:30–3:00pm

Wed 1:30–3:00pm

Fri 1:30–2:30pm

**Allison Tanguay's QCenter Drop-in Hours,** in SMUD 208:

MWF 10am – noon

TuTh 1pm – 4pm

**Math Fellow Drop-in Hours,** in SMUD 208:

Sun 6:00–7:30pm (Kevin)

Mon 7:30–9:00pm (Claire)

Tue 8:30–10:00pm (Aidan)

Wed 7:30–9:00pm (Claire)

Thu 8:30–10:00pm (Aidan)

Fri 6:00–7:30pm (Kevin)

Also, you may email me any time at [rlbenedetto@amherst.edu](mailto:rlbenedetto@amherst.edu)