

Homework #3Due **Friday, February 7** in Gradescope by **11:59 pm ET****READ** Sections 3–4 in Saracino

- WATCH** 1. Required: Video 6: Powers of Group Elements (21:27)
2. Required: Video 7: Greatest Common Divisors (5:22)

WRITE AND SUBMIT solutions to the following problems.**Problem 1.** (8 points) Saracino, Section 3, Problem 3.9:Let $(G, *)$ be a group. Prove that $(G, *)$ is abelian if and only if:

$$(x * y)^{-1} = x^{-1} * y^{-1} \quad \text{for all } x, y \in G.$$

Problem 2. (10 points) Saracino, Section 3, Problem 3.11:Let $(G, *)$ be a group such that $x^2 = e$ for all $x \in G$. [Notation: x^2 means $x * x$.] Prove that G is abelian.**Problem 3.** (10 points) Saracino, Section 3, Problem 3.12:Let $(G, *)$ be a group. Prove that G is abelian if and only if $(x * y)^2 = x^2 * y^2$ for all $x, y \in G$.**Problem 4.** (10 points) Saracino, Section 4, Problem 4.8:It is a fact [which you may assume without proof] that the set $2\mathbb{Z}$ of even integers forms a group under addition. Is this group cyclic? Prove or disprove.

[Note from RLB: Saracino doesn't explicitly say "Prove or disprove," but every time a mathematician asks you a question like that, they want you to justify your answer, i.e., prove it.]

Problem 5. (15 points) Saracino, Section 4, Problem 4.9: Prove that the group $(\mathbb{Q}_{>0}, \cdot)$ is not cyclic.[Note from RLB: Saracino writes \mathbb{Q}^+ for what I'm calling $\mathbb{Q}_{>0}$ here, i.e., $\mathbb{Q} \cap (0, \infty)$.]**Problem 6.** (5 points) [Not from Saracino, but may be useful on HW 4.]Fix a positive integer $n \geq 1$. Let $x_1, x_2, y_1, y_2 \in \mathbb{Z}$ be integers such that $x_1 \equiv x_2 \pmod{n}$ and $y_1 \equiv y_2 \pmod{n}$. Prove that $x_1 y_1 \equiv x_2 y_2 \pmod{n}$.**Problem 7.** (8 points) [Not from Saracino, but may be useful on HW 4.]Let G be a group, and let $a, x \in G$. Use induction to prove that for any integer $n \geq 1$, we have $(x a x^{-1})^n = x a^n x^{-1}$.

(Optional Challenges and Office Hour Information on next page)

Optional Challenges (do NOT hand in): Saracino Problems 3.14, 4.25

Questions? You can ask in:

Class:

Section 01: MWF 9:00–9:50am, SMUD 014

Section 02: MWF 11:00–11:50am, SMUD 205

My office hours: in my office (SMUD 406):

Tue 1:30–3:00pm

Wed 1:30–3:00pm

Fri 1:30–2:30pm

Allison Tanguay's QCenter Drop-in Hours, in SMUD 208:

MWF 10am – noon

TuTh 1pm – 4pm

Math Fellow Drop-in Hours, in SMUD 208:

Sun 6:00–7:30pm (Kevin)

Mon 7:30–9:00pm (Claire)

Tue 7:30–9:00pm (Aidan)

Wed 7:30–9:00pm (Claire)

Thu 7:30–9:00pm (Aidan)

Fri 6:00–7:30pm (Kevin)

Also, you may email me any time at rlbenedetto@amherst.edu