

Homework #21 (Last one!)Due **Tuesday, May 6** in Gradescope by **11:59 pm ET****READ** Sections 20–21 in Saracino

- WATCH** 1. Optional: Video 42: Norms, Units, and (non)-UFDs (24:19)
2. Optional: Video 43: Every PID is a UFD (38:30)
3. Optional: Video 44: $\mathbb{Z}[i]$ is a PID (36:16)

WRITE AND SUBMIT solutions to the following problems.**Problem 1.** (12 points) Saracino, Section 20, Problem 20.1.

Let p be a prime. Prove that $\mathbb{F}_p[X]/\langle X^2 + 1 \rangle$ is a field if and only if the equation $x^2 \equiv -1 \pmod{p}$ has no solution (modulo p).

Problem 2. (10 points) Saracino, Section 20, Problem 20.4.

Let $K = \{0, 1, \alpha, \alpha + 1\}$ be the four-element field constructed in Example 1 of Section 20 (pages 206–207), where I have written α for the element Saracino denotes \overline{X} . Write the polynomial $X^2 + X + 1$ as a product of factors of degree 1 in $K[X]$.

Problem 3. (18 points) Saracino, Section 20, variant of Problem 20.7(a).

Construct a field of 8 elements using Theorems 17.7, 19.8, and 20.2.

[As always, don't forget to verify that the field you construct actually is a field, and that it actually has exactly 8 elements.]

Problem 4. (15 points) Saracino, Section 20, variant of Problem 20.7(b).

Construct a field of 9 elements using Theorems 17.7, 19.8, and 20.2.

[As always, don't forget to verify that the field you construct actually is a field, and that it actually has exactly 9 elements.]

Optional Challenges (do NOT hand in): Saracino Problems 20.9, 20.10, 21.6, 21.10, 21.23

Questions? You can ask in:

Class:

Section 01: MWF 9:00–9:50am, SMUD 014

Section 02: MWF 11:00–11:50am, SMUD 205

My office hours: in my office (SMUD 406):

Tue 1:30–3:00pm

Wed 1:30–3:00pm

Fri 1:30–2:30pm

Allison Tanguay's QCenter Drop-in Hours, in SMUD 208:

MWF 10am – noon

TuTh 1pm – 4pm

Math Fellow Drop-in Hours, in SMUD 208:

Sun 6:00–7:30pm (Kevin)

Mon 7:30–9:00pm (Claire)

Tue 8:30–10:00pm (Aidan)

Wed 7:30–9:00pm (Claire)

Thu 8:30–10:00pm (Aidan)

Fri 6:00–7:30pm (Kevin)

Also, you may email me any time at rlbenedetto@amherst.edu