Math 350, Spring 2025 Professor Rob Benedetto

Homework #2

Due **Tuesday, February 4** in Gradescope by **11:59 pm ET**

READ Sections 2–3 in Saracino

WATCH 1. Required: Video 3: Proving G is a Group (12:42)

2. Required: Video 4: Adding modulo n (15:00)

3. Optional For Fun: Video 5: One-Sided Identities or Inverses (12:37)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (7 points) Saracino, Section 2, Problem 2.4(a,b):
Note: I use the notation C_n for the group Saracino calls $\mathbb{Z}_n$. You may use either notation. Just be aware that they mean the same thing. The problem says this:

Write down the multiplication tables for the following groups:

(a): $(C_4, \oplus)$ (b): $(C_5, \oplus)$

Problem 2. (6 points) Saracino, Section 2, Problem 2.5:
The problem asks whether $S = \{a, b, c\}$, with operation $*$ given by the multiplication table below, is a group.
If you decide the answer is yes, give a short explanation, but not a formal proof. If you decide the answer is no, give one explicit example of one axiom or combination of axioms failing.

$*$	a	b	c
a	a	b	c
b	b	b	c
c	c	c	c

Is $(S, *)$ a group? [And say why or why not as above.]

Problem 3. (14 points) Saracino, Section 2, Problem 2.8:
Let G be the set of all functions $f : \mathbb{R} \to \mathbb{R}$ which have the property that $f(x) \neq 0$ for all $x \in \mathbb{R}$. Define the product of $f, g \in G$ by

$$(f \times g)(x) = f(x)g(x) \quad \text{for all } x \in \mathbb{R}.$$

Is $(G, \times)$ a group? Prove or disprove.

Problem 4. (14 points) Saracino, Section 2, Problem 2.11:
Let G be the set of all 2×2 matrices $\begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix}$, where $a, b \in \mathbb{R}$ and $a, b \in \mathbb{R} \smallsetminus \{0\}$ are nonzero real numbers. Prove that G forms a group under matrix multiplication.

(continued next page)

Problem 5. (8 points) Saracino, Section 3, Problem 3.3:
Find elements $A, B, C \in GL(2, \mathbb{R})$ such that $AB = BC$ but $A \neq C$.

Problem 6. (8 points) Saracino, Section 3, Problem 3.4:
Let $(G, *)$ be a group, and let $g \in G$. Suppose that there is (at least) one element $x \in G$ such that $x * g = x$. Prove that $g = e$.

Optional Challenges (do NOT hand in): Saracino Problems 2.10, 2.13, 3.7

Questions? You can ask in:

Class:
Section 01: MWF 9:00–9:50am, SMUD 014
Section 02: MWF 11:00–11:50am, SMUD 205

My office hours: in my office (SMUD 406):
Tue 1:30–3:00pm
Wed 1:30–3:00pm
Fri 1:30–2:30pm

Allison Tanguay's QCenter Drop-in Hours, in SMUD 208:
MWF 10am – noon
TuTh 1pm – 4pm

(Math Fellow office hours will start soon, too!)

Also, you may email me any time at `rlbenedetto@amherst.edu`