Homework #9

Due **Friday, February 27** in Gradescope by **11:59 pm ET**

READ Section 8 in Saracino and the **Orders of Permutations** handout

WATCH Required: Video 18: Cycle Notation Proof (11:30)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (12 points) Saracino, Section 8, Problem 8.2:
Write each permutation as a product of disjoint cycles, and then as a product of transpositions. Determine whether each permutation is even or odd.

(a): $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 1 & 4 & 2 & 5 \end{pmatrix}$ (b): $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 1 & 3 & 5 \end{pmatrix}$

(c): $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 3 & 4 & 1 & 2 \end{pmatrix}$ (d): $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 1 & 2 & 3 \end{pmatrix}$

Problem 2. (16 points) Saracino, Section 8, Problem 8.5:
Write down all the elements of S_4 [in disjoint cycle notation], and indicate which ones are in A_4. Check your results against Theorem 8.5.

Problem 3. (6 points) Saracino, Section 8, Problems 8.10(b) and 8.3(c):
Let $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 \\ 6 & 7 & 5 & 9 & 8 & 4 & 11 & 3 & 1 & 12 & 2 & 10 \end{pmatrix} \in S_{12}$.
(a): Find the order of σ. [This is problem 8.10(b).]
(b): Decide whether σ is even or odd. [This is problem 8.3(c).]

Problem 4. (6 points) Saracino, Section 8, Problem 8.11(a):
Give an example of two elements $f, g \in S_9$ such that $o(f) = o(g) = 5$ and $o(fg) = 9$.
[And of course, verify that your example has all these properties.]

Problem 5. (12 points) Saracino, Section 8, Problem 8.12:
Does A_6 have an element of order 6? Does A_7? [As always, justify your answers.]

Problem 6. (12 points) Saracino, Section 8, Problem 8.16:
Let X be a set and let $Y \subseteq X$ be a subset. Prove that $\{f \in S_X \mid \forall y \in Y, \text{ we have } f(y) = y\}$ is a subgroup of S_X.

Optional Challenges (do NOT hand in): Saracino Problems 8.7, 8.20

(Office Hour Information on next page)

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
Mon 2:00–3:30pm
Tue 1:30–3:15pm
Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
Drop-in Hours: Mon-to-Fri, 9am – noon.
Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun	7:30–9:00pm	(Javier)
Mon	6:00–7:30pm	(Megan)
Tue	6:00–7:30pm	(Torin)
Tue	7:30–9:00pm	(Javier)
Wed	7:30–9:00pm	(Megan)
Thu	6:00–7:30pm	(Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`