Homework #8

Due **Tuesday, February 24** in Gradescope by **11:59 pm ET**

READ Sections 7–8 in Saracino and the **Functions** handout

WATCH 1. Optional: Video 16: Sun Tzu's Theorem (10:27)

2. Optional: Video 17: Rigorous Definition of Function (7:02)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (14 points) Saracino, Section 7, Problem 7.1(e,g,i):
In each example below f is given either as a rule or as a set of ordered pairs. In each case, determine whether f is a function from S to T. When it is, determine whether it is one-to-one, and whether it is onto.

e. $S = T = \mathbb{R}$, $f(x) = x^3$

g. $S = T = \mathbb{R}$, $f(x) = 1/x$

i. $S = T = \mathbb{Z}_{\geq 1}$, $f(x) = \begin{cases} 1 & \text{if } x = 1, \\ x - 1 & \text{if } x > 1 \end{cases}$

Problem 2. (10 points) Saracino, Section 7, Problem 7.4:
Let $S = T = \{\text{polynomials with real coefficients}\}$, and define a function $D : S \to T$ mapping each polynomial f to its derivative f'. Is this function one-to-one? Is it onto? (For both questions, don't forget to prove your answer.)

Problem 3. (9 points) Saracino, Section 7, Problem 7.8:
Let G be a group, and let $f(x) = x^{-1}$ for all $x \in G$. Is f a function from G to G? If so, is it one-to-one? Is it onto? Prove your answers.

Problem 4. (12 points) Saracino, Section 7, Problem 7.10(a):
Prove that $f : S \to T$ is one-to-one if and only if there exists a function $g : T \to S$ such that $g \circ f = \mathrm{id}_S$. [Note from RLB: you may assume here that $S \neq \varnothing$.]

Problem 5. (10 points) Saracino, Section 7, Problem 7.12(a):
Let $f : S \to T$. For any subset $A \subseteq S$, define $f(A) = \{f(s) \mid s \in A\}$, which is a subset of T. If $A, B \subseteq S$ are subsets, prove that $f(A \cup B) = f(A) \cup f(B)$.

Problem 6. (12 points) Saracino, Section 7, Problem 7.12(b):
With notation as in the previous problem, prove that $f(A \cap B) \subseteq f(A) \cap f(B)$. Construct an example where the inclusion is proper, i.e., where $f(A \cap B) \subsetneq f(A) \cap f(B)$.

(Problems continue on next page)

Problem 7. (6 points) Saracino, Section 8, Problem 8.1: Compute the following multiplications in S_6:

(a): $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 6 & 1 & 4 & 2 & 5 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 4 & 3 & 2 & 1 & 6 \end{pmatrix}$ (b): $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 2 & 5 & 4 & 1 & 6 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 1 & 2 & 3 \end{pmatrix}$

(c): $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 2 & 4 & 6 & 1 & 3 & 5 \end{pmatrix} \circ \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 3 & 4 & 1 & 2 \end{pmatrix}$

Optional Challenges (do NOT hand in): Saracino Problems 7.5, 7.11

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
Mon 2:00–3:30pm
Tue 1:30–3:15pm
Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
Drop-in Hours: Mon-to-Fri, 9am – noon.
Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun	7:30–9:00pm	(Javier)
Mon	6:00–7:30pm	(Megan)
Tue	6:00–7:30pm	(Torin)
Tue	7:30–9:00pm	(Javier)
Wed	7:30–9:00pm	(Megan)
Thu	6:00–7:30pm	(Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`