Homework #7
Due **Friday, February 20** in Gradescope by **11:59 pm ET**

READ Section 6 in Saracino and the **Subgroups of Cyclic Groups** handout

WATCH 1. Required: Video 15: Products of Cyclic Groups (17:33)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (10 points) Saracino, Section 5, Problem 5.23:
Let G be a group, and let $g \in G$. Define the **centralizer** of g in G to be the subset
$$Z(g) = \{x \in G \mid xg = gx\}.$$
Prove that $Z(g)$ is a subgroup of G.

Problem 2. (10 points) Saracino, Section 5, Problem 5.25:
Let G be a group and let $a \in G$. Let H be a subgroup of G, and define $aHa^{-1} = \{aha^{-1} \mid h \in H\}$, i.e., the set of all elements of G that are of the form aha^{-1} with $h \in H$. Prove that aHa^{-1} is a subgroup of G.

Problem 3. (8 points) Saracino, Section 6, Problem 6.1(c,d): Calculate the order of the element:
   c. $(8, 6, 4)$ in $C_{18} \times C_9 \times C_8$
   d. $(8, 6, 4)$ in $C_9 \times C_{17} \times C_{10}$

Problem 4. (9 points) Saracino, Section 6, Problem 6.2(a,b,d): Which of the following groups are cyclic? [And why?]
   a. $C_{12} \times C_9$
   b. $C_{10} \times C_{85}$
   d. $C_{22} \times C_{21} \times C_{65}$

Problem 5. (8 points) Saracino, Section 6, Problem 6.6:
Prove that $G_1 \times G_2 \times \cdots \times G_n$ is abelian if and only if each G_i is abelian.

Problem 6. (10 points) Saracino, Section 6, Problem 6.8:
Construct a group of order 81 with the property that every element except the identity has order 3. Of course, don't forget to verify (probably using the theorems of Section 6) that your answer is in fact a group and that it has the desired properties.

Optional Challenges (do NOT hand in): Saracino Problems 6.3, 6.7, 6.10, 6.14

(Office Hour Information on next page)

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
    Mon 2:00–3:30pm
    Tue 1:30–3:15pm
    Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
    Drop-in Hours: Mon-to-Fri, 9am – noon.
    Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:
    Sun 7:30–9:00pm   (Javier)
    Mon 6:00–7:30pm   (Megan)
    Tue 6:00–7:30pm   (Torin)
    Tue 7:30–9:00pm   (Javier)
    Wed 7:30–9:00pm   (Megan)
    Thu 6:00–7:30pm   (Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`