

**Homework #6**Due **Tuesday, February 17** in Gradescope by **11:59 pm ET****READ** Sections 5–6 in Saracino and the **Proof of the Subgroup Theorem** handout**WATCH** 1. Required: Video 13: Subgroups of Cyclic Groups (15:12)

2. Optional: Video 14: Combining Infinitely Many Groups (20:32)

**WRITE AND SUBMIT** solutions to the following problems.**Problem 1.** (4 points) [Not from Saracino, but useful on the next problem]Let  $G$  be a group, let  $H \subseteq G$  be a subgroup, and let  $x \in G$ . If  $x \in H$ , prove that  $\langle x \rangle \subseteq H$ .[**Note** from RLB: Saracino calls essentially this fact “obvious” near the top of page 46. I still think it deserves a (short) proof, though.]**Problem 2.** (20 points) Saracino, Section 5, Problem 5.5(a):Find all the subgroups of  $Q_8$ . Make sure to explain why each set in your list actually is a subgroup, **and** prove that your list must be complete.You may assume the following fact: that all the subgroups of  $Q_8$  have order dividing 8, i.e., order 1, 2, 4, or 8. (This is due an upcoming result known as **Lagrange’s Theorem**.)[**Note 1** from RLB: Besides the 1,2,4,8 fact that you may assume, you may find Problem 1 above useful. As a hint, it should turn out that there are six subgroups: one each of orders 1, 2, and 8, and three of order 4.][**Note 2** from RLB: the (unassigned) part (b) in the book asks you to prove that  $Q_8$  is a nonabelian group, but all of its proper subgroups are cyclic. This is just a matter of noting that each of the subgroups you listed, besides  $Q_8$  itself, is cyclic; but still, it’s an interesting observation.]**Problem 3.** (9 points) Saracino, Section 5, Problem 5.7:Let  $G = \langle x \rangle$  be a cyclic group of finite order  $n$ . Show that for any integer  $m \in \mathbb{Z}$ , the element  $x^m$  is a generator of  $G$  if and only if  $(m, n) = 1$ .

[Saracino has more commentary, and you should read that, but that commentary is not part of what you need to do to complete this problem.]

**Problem 4.** (6 points) Saracino, Section 5, Problem 5.8:Let  $G = \langle x \rangle$  be a cyclic group of order 144. How many elements are there in the subgroup  $\langle x^{26} \rangle$ ?**Problem 5.** (10 points) Saracino, Section 5, Problem 5.11:Let  $G$  be an abelian group, and let  $n \geq 1$  be a positive integer. Let  $H = \{x \in G \mid x^n = e\}$ .[Or as Saracino phrases it, let  $H$  be the subset of  $G$  consisting of all  $x \in G$  such that  $x^n = e$ .]Prove that  $H$  is a subgroup of  $G$ .

(problem list continues on next page)

**Problem 6.** (8 points) Saracino, Section 5, Problem 5.14:

Let  $H, K$  be subgroups of a group  $G$ . Prove that  $H \cap K$  is also a subgroup of  $G$ .

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**Optional Challenges (do NOT hand in):** Saracino Problems 5.26, 5.28

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**Questions?** You can ask in:

**Class:** MWF 11:35am – 12:25pm, SMUD 207

**My office hours:** in my office (SMUD 406):

Mon 2:00–3:30pm

Tue 1:30–3:15pm

Fri 1:00–2:00pm

**David Metacarpa’s QCenter Hours,** in SMUD 208:

Drop-in Hours: Mon-to-Fri, 9am – noon.

Also available by appointment in the afternoons

**Math Fellow Drop-in Hours,** in SMUD 206:

Sun 7:30–9:00pm (Javier)

Mon 6:00–7:30pm (Megan)

Tue 6:00–7:30pm (Torin)

Tue 7:30–9:00pm (Javier)

Wed 7:30–9:00pm (Megan)

Thu 6:00–7:30pm (Torin)

Also, you may email me any time at [rlbenedetto@amherst.edu](mailto:rlbenedetto@amherst.edu)