Math 350, Spring 2026 Professor Rob Benedetto

Homework #5

Due **Friday, February 13** in Gradescope by **11:59 pm ET**

READ Section 5 in Saracino

WATCH 1. Required: Video 10: The $mx + ny$ Theorem (13:51)

2. Optional: Video 11: Another $mx + ny$ Proof (14:06)

3. Required: Video 12: Proof of the Order Theorem (17:23)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (17 points) Saracino, Section 5, Problem 5.1(a,c,d):
In each case, determine whether or not H is a subgroup of G. [Note from RLB: If yes, prove it using the three conditions of Theorem 5.1: nonempty, closed under $*$, and closed under inverses. If no, give one example of one of these conditions failing.]

(a) $G = (\mathbb{R}, +)$, $H = \mathbb{Q}$

(c) $G = (\mathbb{Z}, +)$, $H = \mathbb{Z}^+$, i.e., $H = \{n \in \mathbb{Z} \mid n > 0\}$

(d) $G = (\mathbb{Q}^\times, \cdot)$, $H = \mathbb{Q}^+$, i.e., $H = \{x \in \mathbb{Q} \mid x > 0\}$

Problem 2. (12 points) Saracino, Section 5, Problem 5.1(e,f):
In each case, determine whether or not H is a subgroup of G. (As on problem 1: If yes, prove it using the three conditions of Theorem 5.1: nonempty, closed under $*$, and closed under inverses. If no, give one example of one of these conditions failing.)

(e) $G = (C_8, \oplus)$, $H = \{0, 2, 4\}$

(f) $G = (\mathbb{R}^2, +)$, i.e., ordered pairs of real numbers under coordinatewise addition, and $H = \{(a, b) \in \mathbb{R}^2 \mid b = -a\}$

Problem 3. (8 points) Saracino, Section 5, Problem 5.2:
Let $G = \{\text{functions } f : \mathbb{R} \to \mathbb{R}\}$ be the group of real-valued functions on $\mathbb{R}$, under addition of functions. Let $H = \{f \in G \mid f \text{ is differentiable}\}$. Show that H is a subgroup of G.

Problem 4. (10 points) Saracino, Section 5, Problem 5.3: Let H be the set of elements $\begin{bmatrix} a & b \\ c & d \end{bmatrix}$ of $GL(2, \mathbb{R})$ such that $ad - bc = 1$. Prove that H is a subgroup of $GL(2, \mathbb{R})$.
[Note: H is called $SL(2, \mathbb{R})$, the *special linear* group of degree 2 over $\mathbb{R}$.]

Problem 5. (12 points) Saracino, Section 5, Problem 5.4(a,b):

(a) How many subgroups does $(C_{18}, \oplus)$ have? What are they?

(b) How many subgroups does $(C_{35}, \oplus)$ have? What are they?

Problem 6. (6 points) Saracino, Section 5, Problem 5.10:
Prove that every subgroup of an abelian group is abelian.

(Optional Challenges and Office Hour Information on next page)

Optional Challenges (do NOT hand in): Saracino Problems 5.9, 5.16, 5.17

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
Mon 2:00–3:30pm
Tue 1:30–3:15pm
Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
Drop-in Hours: Mon-to-Fri, 9am – noon.
Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun	7:30–9:00pm	(Javier)
Mon	6:00–7:30pm	(Megan)
Tue	6:00–7:30pm	(Torin)
Tue	7:30–9:00pm	(Javier)
Wed	7:30–9:00pm	(Megan)
Thu	6:00–7:30pm	(Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`