

Homework #4Due **Tuesday, February 10** in Gradescope by **11:59 pm ET****READ** Section 4 in Saracino**WATCH** 1. Required: Video 8: The Euclidean Algorithm (13:31)2. Optional: Video 9: Proof of the Euclidean Algorithm (10:38)**WRITE AND SUBMIT** solutions to the following problems.**Problem 1.** (8 points) Saracino, Section 4, Problem 4.5:Let G be a group, and let $x \in G$ be an element of order 18. Find the orders of $x^2, x^3, x^4, x^5, x^{12}$.**Problem 2.** (10 points) Saracino, Section 4, Problem 4.6:List all the elements of (C_{45}, \oplus) that are of order 15. As always, justify your answer.[Note from RLB: recall I write C_{45} for what Saracino calls \mathbb{Z}_{45} .]**Problem 3.** (15 points) Saracino, Section 4, Problem 4.10(a):Let $G = \{1, 2, 3, 4, 5, 6\}$ and define an operation \odot on G by $a \odot b = \overline{ab}$, the remainder of ab (mod 7). For instance, $2 \odot 4 = \overline{8} = 1$, and $5 \odot 6 = \overline{30} = 2$. Prove that (G, \odot) is a group.[*Suggestion:* for associativity, use the result of HW 3, Problem 6.]**Problem 4.** (8 points) Saracino, Section 4, Problem 4.10(b):Is the group (G, \odot) above cyclic? Prove or disprove.**Problem 5.** (12 points) Saracino, Section 4, Problem 4.20:Let G be a group and $a \in G$. We say an element $b \in G$ is a *conjugate* of a if there exists $x \in G$ such that $b = xax^{-1}$. Show that any conjugate of a has the same order as a .*Suggestions* for problem 5: For one thing: use HW 3, Problem 7.For another thing: to show $o(g) = o(h)$, start by defining two sets $S = \{n \geq 1 \mid g^n = e\}$ and $T = \{n \geq 1 \mid h^n = e\}$, and then prove that the sets S and T are equal. Then $o(g)$ is the smallest element of S , or ∞ if $S = \emptyset$, and similarly for $o(h)$ for T .**Problem 6.** (10 points) Saracino, Section 4, Problem 4.21: Let G be a group and $x, y \in G$. Prove that $o(xy) = o(yx)$.

Optional Challenges (do NOT hand in): Saracino Problems 4.22, 4.23

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):

Mon 2:00–3:30pm

Tue 1:30–3:15pm

Fri 1:00–2:00pm

David Metacarpa’s QCenter Hours, in SMUD 208:

Drop-in Hours: Mon-to-Fri, 9am – noon.

Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun 7:30–9:00pm (Javier)

Mon 6:00–7:30pm (Megan)

Tue 6:00–7:30pm (Torin)

Tue 7:30–9:00pm (Javier)

Wed 7:30–9:00pm (Megan)

Thu 6:00–7:30pm (Torin)

Also, you may email me any time at rlbenedetto@amherst.edu