

**Homework #3**Due **Friday, February 6** in Gradescope by **11:59 pm ET****READ** Sections 3–4 in Saracino**WATCH** 1. Required: Video 6: Powers of Group Elements (21:27)2. Required: Video 7: Greatest Common Divisors (5:22)**WRITE AND SUBMIT** solutions to the following problems.

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**Problem 1.** (8 points) Saracino, Section 3, Problem 3.9:Let  $(G, *)$  be a group. Prove that  $(G, *)$  is abelian if and only if:

$$(x * y)^{-1} = x^{-1} * y^{-1} \quad \text{for all } x, y \in G.$$

**Problem 2.** (10 points) Saracino, Section 3, Problem 3.11:Let  $(G, *)$  be a group such that  $x^2 = e$  for all  $x \in G$ . [Notation:  $x^2$  means  $x * x$ .] Prove that  $G$  is abelian.**Problem 3.** (10 points) Saracino, Section 3, Problem 3.12:Let  $(G, *)$  be a group. Prove that  $G$  is abelian if and only if  $(x * y)^2 = x^2 * y^2$  for all  $x, y \in G$ .**Problem 4.** (10 points) Saracino, Section 4, Problem 4.8:It is a fact [which you may assume without proof] that the set  $2\mathbb{Z}$  of even integers forms a group under addition. Is this group cyclic? Prove or disprove.

[Note from RLB: Saracino doesn't explicitly say "Prove or disprove," but every time a mathematician asks you a question like that, they want you to justify your answer, i.e., prove it.]

**Problem 5.** (15 points) Saracino, Section 4, Problem 4.9: Prove that the group  $(\mathbb{Q}_{>0}, \cdot)$  is not cyclic.[Note from RLB: Saracino writes  $\mathbb{Q}^+$  for what I'm calling  $\mathbb{Q}_{>0}$  here, i.e.,  $\mathbb{Q} \cap (0, \infty)$ .]**Problem 6.** (5 points) [Not from Saracino, but may be useful on HW 4.]Fix a positive integer  $n \geq 1$ . Let  $x_1, x_2, y_1, y_2 \in \mathbb{Z}$  be integers such that  $x_1 \equiv x_2 \pmod{n}$  and  $y_1 \equiv y_2 \pmod{n}$ . Prove that  $x_1 y_1 \equiv x_2 y_2 \pmod{n}$ .**Problem 7.** (8 points) [Not from Saracino, but may be useful on HW 4.]Let  $G$  be a group, and let  $a, x \in G$ . Use induction to prove that for any integer  $n \geq 1$ , we have  $(x a x^{-1})^n = x a^n x^{-1}$ .

(Optional Challenges and Office Hour Information on next page)

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**Optional Challenges (do NOT hand in):** Saracino Problems 3.14, 4.25

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**Questions?** You can ask in:

**Class:** MWF 11:35am – 12:25pm, SMUD 207

**My office hours:** in my office (SMUD 406):

Mon 2:00–3:30pm

Tue 1:30–3:15pm

Fri 1:00–2:00pm

**David Metacarpa’s QCenter Hours,** in SMUD 208:

Drop-in Hours: Mon-to-Fri, 9am – noon.

Also available by appointment in the afternoons

**Math Fellow Drop-in Hours,** in SMUD 206:

Sun 7:30–9:00pm (Javier)

Mon 6:00–7:30pm (Megan)

Tue 6:00–7:30pm (Torin)

Tue 7:30–9:00pm (Javier)

Wed 7:30–9:00pm (Megan)

Thu 6:00–7:30pm (Torin)

Also, you may email me any time at [rlbenedetto@amherst.edu](mailto:rlbenedetto@amherst.edu)