Math 350, Spring 2026 Professor Rob Benedetto

Homework #21 (Last one!)

Due **Tuesday, May 5** in Gradescope by **11:59 pm ET**

READ Sections 20–21 in Saracino

WATCH 1. Optional: Video 43: Norms, Units, and (non)-UFDs (24:19)

2. Optional: Video 44: Every PID is a UFD (38:30)

3. Optional: Video 45: $\mathbb{Z}[i]$ is a PID (36:16)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (12 points) Saracino, Section 20, Problem 20.1.
Let p be a prime. Prove that $\mathbb{F}_p[X]/\langle X^2+1\rangle$ is a field if and only if the equation $x^2 \equiv -1 \pmod{p}$ has no solution (modulo p).

Problem 2. (10 points) Saracino, Section 20, Problem 20.4.
Let $K = \{0, 1, \alpha, \alpha+1\}$ be the four-element field constructed in Example 1 of Section 20 (pages 206–207), where I have written α for the element Saracino denotes $\overline{X}$. Write the polynomial $X^2 + X + 1$ as a product of factors of degree 1 in $K[X]$.

Problem 3. (18 points) Saracino, Section 20, variant of Problem 20.7(a).
Construct a field of 8 elements using Theorems 17.7, 19.8, and 20.2.

[As always, don't forget to verify that the field you construct actually is a field, and that it actually has exactly 8 elements.]

Problem 4. (15 points) Saracino, Section 20, variant of Problem 20.7(b).
Construct a field of 9 elements using Theorems 17.7, 19.8, and 20.2.

[As always, don't forget to verify that the field you construct actually is a field, and that it actually has exactly 9 elements.]

Optional Challenges (do NOT hand in): Saracino Problems 20.9, 20.10, 21.6, 21.10, 21.23

(Office Hour Information on next page)

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
Mon 2:00–3:30pm
Tue 1:30–3:15pm
Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
Drop-in Hours: Mon-to-Fri, 9am – noon.
Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun	7:30–9:00pm	(Javier)
Mon	6:00–7:30pm	(Megan)
Tue	6:00–7:30pm	(Torin)
Tue	7:30–9:00pm	(Javier)
Wed	7:30–9:00pm	(Megan)
Thu	6:00–7:30pm	(Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`