

Homework #18Due **Friday, April 24** in Gradescope by **11:59 pm ET****READ** Sections 17–18 in Saracino and the (optional!) **Theorem on Norms** handout**WATCH** 1. Required: Video 35: Maximal Ideals and Fields (13:41)

2. Optional: Video 36: The Minimal Subfield (17:29)

3. Optional: Video 37: The First Isomorphism Theorem for Rings (7:20)

WRITE AND SUBMIT solutions to the following problems.**Problem 1.** (17 points) Saracino, Section 17, Problem 17.9:Let $R = \{q \in \mathbb{Q} \mid q = a/b \text{ with } a, b \in \mathbb{Z} \text{ and } b \text{ is odd}\}$. [You may take my word for it that R is a commutative ring with unity.] Prove that R has a unique maximal ideal.

[Hint from RLB: it's a principal ideal.]

Problem 2. (8 points) Saracino, Section 17, Problem 17.13:Let I be an ideal of a ring R . Prove that the distributive laws hold in R/I .**Problem 3.** (8 points) Saracino, Section 17, Problem 17.14:Let R be a ring and I an ideal of R .

- (a) If R is commutative, prove that R/I is commutative.
- (b) If R has unity, prove that R/I has unity.

Problem 4. (14 points) Saracino, Section 17, Problem 17.22(a,b):Let R be a commutative ring and X a subset of R . The **annihilator** of X is

$$\text{Ann}(X) = \{r \in R \mid rx = 0 \text{ for every } x \in X\}.$$

- (a) Prove that $\text{Ann}(X)$ is an ideal of R .
- (b) Let $R = \mathbb{Z}/12\mathbb{Z}$. Find $\text{Ann}(\{2\})$.

[Note from RLB: on part (b), by 2, of course, Saracino means the element $12\mathbb{Z} + 2$ of the ring $\mathbb{Z}/12\mathbb{Z}$. This is because it is quite common to denote the unity element of a ring by 1, in which case the element $1 + 1$ is similarly named 2, as Saracino and I have called it here.]

Problem 5. (14 points) Saracino, Section 17, Problem 17.33:Let R be a ring, and let I and J be ideals of R . Define $I + J = \{x + y \mid x \in I, y \in J\}$.

- (a) Prove that $I + J$ is an ideal of R .
- (b) Let $R = \mathbb{Z}$. Find $6\mathbb{Z} + 14\mathbb{Z}$. [And prove your answer, of course.]

Problem 6. (14 points) Saracino, Section 18, Problem 18.1(b,c,e):

Which of the following are ring homomorphisms? [Prove your answers, of course]

- (b) $\varphi : \mathbb{C} \rightarrow \mathbb{C}$ by $\varphi(a + bi) = a - bi$
- (c) $\varphi : \mathbb{C} \rightarrow \mathbb{R}$ by $\varphi(a + bi) = a$
- (e) Let R be the ring of polynomials with real coefficients, and let $\varphi : R \rightarrow R$ by $\varphi(p(x)) = p'(x)$, the derivative of $p(x)$.

(Optional Challenges and Office Hour Information on next page)

Optional Challenges (do NOT hand in): Saracino Problems 17.27, 17.28(a), 17.30

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):

Mon 2:00–3:30pm

Tue 1:30–3:15pm

Fri 1:00–2:00pm

David Metacarpa’s QCenter Hours, in SMUD 208:

Drop-in Hours: Mon-to-Fri, 9am – noon.

Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun 7:30–9:00pm (Javier)

Mon 6:00–7:30pm (Megan)

Tue 6:00–7:30pm (Torin)

Tue 7:30–9:00pm (Javier)

Wed 7:30–9:00pm (Megan)

Thu 6:00–7:30pm (Torin)

Also, you may email me any time at rlbenedetto@amherst.edu