Math 350, Spring 2026 Professor Rob Benedetto

Homework #15

Due **Friday, April 3** in Gradescope by **11:59 pm ET**

READ Section 12 in Saracino; start Section 13 and the (optional!) **Automorphisms** handout

WATCH Optional: Video 29: Cayley's Theorem (9:52)

WRITE AND SUBMIT solutions to the following problems.

Don't forget to **justify all your claims**. (But a lot of justifications are short!)

Problem 1. (18 points) Saracino, Section 12, Problem 12.4(c,f):
In each case, determine whether or not the two groups are isomorphic. [And prove it!]

(c) (6 points) $(\mathbb{R}^\times, \cdot)$ and $(\mathbb{R}, +)$

(f) (12 points) $(\mathbb{R}^\times, \cdot)$ and $(\mathbb{R}_{>0}, \cdot) \times (C_2, \oplus)$

Problem 2. (14 points) Saracino, Section 12, Problem 12.13: Let $\varphi : G \to H$ be a homomorphism.

(a) If H is abelian and φ is one-to-one, prove that G is abelian.

(b) If G is abelian and φ is onto, prove that H is abelian.

(c) If φ is an isomorphism, prove that G is abelian if and only if H is abelian.

Problem 3. (6 points) Saracino, Section 12, Problem 12.15:
Let $\varphi : G \to H$ be an onto homomorphism. If G is cyclic, prove that H is also cyclic.

Problem 4. (14 points) Saracino, Section 12, Problem 12.21:
Let G be the group $\mathbb{C}^\times$ of nonzero complex numbers under multiplication, and let

$$H = \left\{ \begin{bmatrix} a & b \\ -b & a \end{bmatrix} \middle| \, a, b \in \mathbb{R} \text{ are not both } 0 \right\}.$$

You may take my word for it that H is a subgroup of $GL(2, \mathbb{R})$. Prove that $G \cong H$.

Problem 5. (9 points) Saracino, Section 12, Problem 12.22:
Let G be a group, and let $g \in G$. Define $\varphi : G \to G$ by $\varphi(x) = gxg^{-1}$. Prove that φ is an automorphism of G.
[That is, prove that φ is an isomorphism. After all, "automorphism" means an isomorphism from a group *to itself*.]

Problem 6. (9 points) Saracino, Section 13, Problem 13.1:
Let $\varphi : C_8 \to C_4$ be given by $\varphi(x) =$ remainder of $x \pmod 4$. Find $\ker(\varphi)$. Also, to which familiar group is $C_8/\ker\varphi$ isomorphic? (Prove your answer, of course.)
[*Hint*: For the second question, use the First Homomorphism Theorem]

Optional Challenges (do NOT hand in): Saracino Problems 12.4(j,k), 12.11, 12.17, 12.20

(Office Hour Information on next page)

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
Mon 2:00–3:30pm
Tue 1:30–3:15pm
Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
Drop-in Hours: Mon-to-Fri, 9am – noon.
Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun	7:30–9:00pm	(Javier)
Mon	6:00–7:30pm	(Megan)
Tue	6:00–7:30pm	(Torin)
Tue	7:30–9:00pm	(Javier)
Wed	7:30–9:00pm	(Megan)
Thu	6:00–7:30pm	(Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`