Math 350, Spring 2026 Professor Rob Benedetto

Homework #13

Due **Friday, March 27** in Gradescope by **11:59 pm ET**

READ Section 11 in Saracino and the (optional!) **Group Actions** handout

WATCH 1. Required: Video 25: The Normal Subgroup Theorem (18:28)

2. Optional: Video 26: Another Coset Multiplication Proof (7:31)

3. Required: Video 27: Some Special Group Constructions (11:38)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (8 points) Saracino, Section 11, Problem 11.5: Let G be a group, let $H \subseteq G$ be a subgroup, and let $K \lhd G$. Prove that $H \cap K \lhd H$.

Problem 2. (12 points) Saracino, Section 11, Problem 11.8: Let G be a group, let $N \lhd G$, and let $H \subseteq G$ be any subgroup of G. Define

$$NH = \{nh \mid n \in N \text{ and } h \in H\}.$$

Prove that NH is a subgroup of G.

Problem 3. (12 points) Saracino, Section 11, Problem 11.11 (slight rephrasing): Which of the 10 subgroups of D_4 are normal, and which are not?

[Note from RLB: We already have a complete list of all subgroups of D_4, which Saracino presents in a "subgroup lattice" at the end of Section 8, on page 76. For each of the 10 subgroups H shown there, you need to decide whether H is normal in D_4, and **justify your answer** either way.]

Problem 4. (10 points) Saracino, Section 11, Problem 11.12(b): Let $G = A_4$. Prove that there exists subgroups $H, K \subseteq G$ such that $K \lhd H$ and $H \lhd G$, but K is not normal in G.

Problem 5. (14 points) Saracino, Section 11, Problem 11.27: Let H be a subgroup of G. Define

$$N(H) = \{g \in G \mid gHg^{-1} = H\},$$

which is called the *normalizer* of H in G.

(a) Prove that $N(H)$ is a subgroup of G.

(b) Prove that $H \lhd N(H)$.

(c) Let $K \subseteq G$ be a subgroup such that $H \lhd K$. Prove that $K \subseteq N(H)$.

Problem 6. (8 points) Saracino, Section 11, Problem 11.10:
Let G be a group, let $g \in G$ have finite order m, and let $H \lhd G$. Prove that the order of the element $Hg \in G/H$ is finite and divides m.

Problem 7. (8 points) Saracino, Section 11, Problem 11.18:
Let G be cyclic and let $H \subseteq G$ be a subgroup. Prove that G/H is cyclic.

(Optional Challenges and Office Hour Information on next page)

Optional Challenges (do NOT hand in): Saracino Problems 11.9, 11.16

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
Mon 2:00–3:30pm
Tue 1:30–3:15pm
Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
Drop-in Hours: Mon-to-Fri, 9am – noon.
Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun	7:30–9:00pm	(Javier)
Mon	6:00–7:30pm	(Megan)
Tue	6:00–7:30pm	(Torin)
Tue	7:30–9:00pm	(Javier)
Wed	7:30–9:00pm	(Megan)
Thu	6:00–7:30pm	(Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`