Math 350, Spring 2026 Professor Rob Benedetto

Homework #12

Due **Tuesday, March 24** in Gradescope by **11:59 pm ET**

READ Sections 10–11 in Saracino and the **Groups of Order Six** handout

WATCH 1. Required: Video 23: The Class Equation (31:31)

2. Optional: Video 24: Conjugacy Classes in S_n (32:49)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (8 points) Saracino, Section 10, Problem 10.15:
Let G be a finite group, and let H be a subgroup of G. Let K be a subgroup of H. Prove that $[G:K]=[G:H][H:K]$.

Problem 2. (8 points) Saracino, Section 10, Problem 10.23(a):
Let $S=$ the set of even integers, and let $T=$ the set of odd integers.
Prove that S and T have the same cardinality.

[Note from RLB: that is, define a function $f:S\to T$, and prove that your function is one-to-one and onto.]

Problem 3. (12 points) Saracino, Section 10, Problem 10.25:
Find the conjugacy classes in Q_8, and write down the class equation for Q_8.

Problem 4. (14 points) Saracino, Section 10, Problem 10.28:
Let p be a prime number, and let n be a positive integer. Let G be a group with $|G|=p^n$. Use the class equation to prove that $|Z(G)|$ is divisible by p.

Problem 5. (12 points) Saracino, Section 10, Problem 10.29:
Let p be a prime number and let G be a group such that $|G|=p^2$. Prove that G is abelian.

[Suggestion from RLB: Use the previous problem. Then, using Lagrange's Theorem, for any $a\in G$ with $a\notin Z(G)$, what can you say about the centralizer $Z(a)$?]

Problem 6. (8 points) Saracino, Section 11, Problem 11.1:

Recall that $SL(2,\mathbb{R})=\left\{\begin{bmatrix}a & b\\ c & d\end{bmatrix}\in GL(2,\mathbb{R})\,\middle|\, ad-bc=1\right\}$.

We have already seen that $SL(2,\mathbb{R})$ is a subgroup of $GL(2,\mathbb{R})$. Prove that $SL(2,\mathbb{R})\lhd GL(2,\mathbb{R})$.

Problem 7. (8 points) Saracino, Section 11, Problem 11.2:

Let H be the subgroup of $G=GL(2,\mathbb{R})$ consisting of all matrices $\begin{bmatrix}a & b\\ 0 & d\end{bmatrix}$ such that $ad\neq 0$. Is H a normal subgroup of G? Why or why not?

(Optional Challenges and Office Hour Information on next page)

Optional Challenges (do NOT hand in): Saracino Problems 10.32, 10.33

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
Mon 2:00–3:30pm
Tue 1:30–3:15pm
Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
Drop-in Hours: Mon-to-Fri, 9am – noon.
Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:

Sun	7:30–9:00pm	(Javier)
Mon	6:00–7:30pm	(Megan)
Tue	6:00–7:30pm	(Torin)
Tue	7:30–9:00pm	(Javier)
Wed	7:30–9:00pm	(Megan)
Thu	6:00–7:30pm	(Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`