Homework #11
Due **Friday, March 13** in Gradescope by **11:59 pm ET**

READ $\boxed{\text{Sections 9–10 in Saracino}}$ and the $\boxed{\textbf{Left and Right Cosets } \text{handout}}$

WATCH 1. $\boxed{\text{Optional Video 21: Generating Sets}}$ (10:33)

           2. $\boxed{\text{Required: Video 22: Multiplication mod } n \text{ is a Group}}$ (14:22)

WRITE AND SUBMIT solutions to the following problems.

Problem 1. (15 points) Saracino, Section 9, Problem 9.16(a,b):
Let H be a subgroup of a group G, and define a relation $_H\!\equiv$ on G by $x\,_H\!\equiv y$ iff $x^{-1}y \in H$.
     a. Show that $_H\!\equiv$ is an equivalence relation on G.
     b. Show that the equivalence classes under $_H\!\equiv$ are precisely the left cosets of H in G.

Problem 2. (7 points) Saracino, Section 10, Problem 10.1 (well, part of it, and in my notation):
Let $G = Q_8$. Find $[G : H]$ for $H = \langle -1 \rangle$, and find $[G : K]$ for $K = \langle j \rangle$.
[As always, justify your answers.]

Problem 3. (7 points) Saracino, Section 10, Problem 10.2(b):
Let $G = C_{54}$. Find $[G : H]$ for $H = \langle 24 \rangle$. [As always, justify your answer.]

Problem 4. (8 points) Saracino, Section 10, Problem 10.3(b):
Let $G = C_6 \times C_4$, and let $H = \langle 2 \rangle \times \langle 2 \rangle$. Find $[G : H]$. [As always, justify your answer.]

Problem 5. (8 points) Saracino, Section 10, Problem 10.5: Let G be a group of order 8 that is not cyclic. For every $a \in G$, prove that $a^4 = e$.

Problem 6. (8 points) Saracino, Section 10, Problem 10.6:
Let G be a group, and let $H, K \subseteq G$ be subgroups. Suppose that $|H| = 12$ and $|K| = 5$. Prove that $H \cap K = \{e\}$.

Problem 7. (10 points) Saracino, Section 10, Problem 10.8:
Let G be a group of order p^2, where p is prime. Prove that G must have a subgroup of order p.

Optional Challenges (do NOT hand in): Saracino Problems 9.14, 9.15

(Office Hour Information on next page)

Questions? You can ask in:

Class: MWF 11:35am – 12:25pm, SMUD 207

My office hours: in my office (SMUD 406):
  Mon 2:00–3:30pm
  Tue 1:30–3:15pm
  Fri 1:00–2:00pm

David Metacarpa's QCenter Hours, in SMUD 208:
  Drop-in Hours: Mon-to-Fri, 9am – noon.
  Also available by appointment in the afternoons

Math Fellow Drop-in Hours, in SMUD 206:
  Sun 7:30–9:00pm   (Javier)
  Mon 6:00–7:30pm   (Megan)
  Tue 6:00–7:30pm   (Torin)
  Tue 7:30–9:00pm   (Javier)
  Wed 7:30–9:00pm   (Megan)
  Thu 6:00–7:30pm   (Torin)

Also, you may email me any time at `rlbenedetto@amherst.edu`