

**Homework #10**Due **Tuesday, March 10** in Gradescope by **11:59 pm ET****READ** Sections 8–9 in Saracino**WATCH** 1. Optional: Video 19: Even and Odd Permutations (37:37)

2. Required: Video 20: The Equivalence Class Theorem (16:26)

**WRITE AND SUBMIT** solutions to the following problems.**Problem 1.** (9 points) Saracino, Section 8, Problem 8.14:  
Find  $Z(S_3)$  and  $Z(D_4)$ . [As always, justify your answers.]**Problem 2.** (8 points) Saracino, Section 8, Problem 8.15(b,c):  
With notation as in the book for the elements of  $D_n$ :

- (b) Calculate  $(f^2g)(f^3g)(f^2)$  in  $D_5$ . [And show your work.]  
 (c) Find  $Z(D_5)$ . [And justify your answer.]

**Problem 3.** (14 points) Saracino, Section 9, Problem 9.1:Determine which of the following relations  $R$  on  $\mathbb{Z}$  are equivalence relations (and justify, of course).

- (a)  $a R b$  iff  $a - b \geq 0$                       (b)  $a R b$  iff  $|a| = |b|$   
 (c)  $a R b$  iff  $ab \geq 0$                          (d)  $a R b$  iff  $|a - b| \leq 1$

**Problem 4.** (10 points) Saracino, Section 9, Problem 9.4:Let  $R$  be an equivalence relation on a set  $S$ . Prove that for all  $s_1, s_2 \in S$ , we have:

$$\overline{s_1} = \overline{s_2} \quad \text{iff} \quad s_1 R s_2.$$

[Here,  $\overline{s}$  denotes the equivalence class containing  $s$ , as defined at the top of page 82. In particular, the statement  $\overline{s_1} = \overline{s_2}$  is an equality of *sets*.]**Problem 5.** (8 points) Saracino, Section 9, Problem 9.5: Find the right cosets of  $H$  in  $G = Q_8$  for:

- a.  $H = \langle i \rangle$  (which Saracino denotes  $\langle J \rangle$ )  
 b.  $H = \langle -1 \rangle$  (which Saracino denotes  $\langle -I \rangle$ )

[As always, justify your answers.]

**Problem 6.** (10 points) Section 9, Problem 9.12: Let  $G$  be a group. Define a relation  $R$  on  $G$  by

$$a R b \quad \text{iff} \quad \exists x \in G \text{ such that } a = bx^{-1}.$$

Prove that  $R$  is an equivalence relation on  $G$ .

(Problems continue on next page)

**Problem 7.** (16 points) Variant of Saracino, Section 9, Problem 9.6:

Let  $H_1 = \{e, f^2g\} = \langle f^2g \rangle$  and  $H_2 = \{e, f^2\} = \langle f^2 \rangle$ ,

both of which are subgroups of  $D_4 = \{e, f, f^2, f^3, g, fg, f^2g, f^3g\}$ .

- Find the right cosets of  $H_1$  in  $D_4$ .
- Find the left cosets of  $H_1$  in  $D_4$ .
- Find the right cosets of  $H_2$  in  $D_4$ .
- Find the left cosets of  $H_2$  in  $D_4$ .
- Briefly: what do you notice about the answers in parts (a), (b), as compared with the answers in parts (c), (d)?

[Note from RLB: in your lists of sets in parts (a)–(d), simplify each element of  $D_4$  to one of the eight in the list on page 75:  $D_4 = \{e, f, f^2, f^3, g, fg, f^2g, f^3g\}$ . As always, justify your answers.]

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**Optional Challenges (do NOT hand in):** Saracino Problems 8.15(d), 8.25

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**Questions?** You can ask in:

**Class:** MWF 11:35am – 12:25pm, SMUD 207

**My office hours:** in my office (SMUD 406):

Mon 2:00–3:30pm

Tue 1:30–3:15pm

Fri 1:00–2:00pm

**David Metacarpa's QCenter Hours,** in SMUD 208:

Drop-in Hours: Mon-to-Fri, 9am – noon.

Also available by appointment in the afternoons

**Math Fellow Drop-in Hours,** in SMUD 206:

Sun 7:30–9:00pm (Javier)

Mon 6:00–7:30pm (Megan)

Tue 6:00–7:30pm (Torin)

Tue 7:30–9:00pm (Javier)

Wed 7:30–9:00pm (Megan)

Thu 6:00–7:30pm (Torin)

Also, you may email me any time at [rlbenedetto@amherst.edu](mailto:rlbenedetto@amherst.edu)