Math 345, Fall 2024: Functions of a Complex Variable

MWF 9–9:50 AM, SMUD 204

Webpage: `http://rlbenedetto.people.amherst.edu/math345`
(Also linked from the Math 345 moodle page.)

Instructor: Rob Benedetto
Office: SMUD 406 **Email**: `rlbenedetto@amherst.edu`
Office Hours: Tue, 1:30–3pm; Thu, 1:30–3pm; Fri 2–3pm; or by appointment.

Our Math 345 Fellow (or "TA"): Karen Kang `kkang25@amherst.edu`

Text: Theodore Gamelin, *Complex Analysis*, Springer, New York, 2001.
For Amherst students: available through the Amherst College Textbook program.
(Five-College students must obtain the book on own.)

Exams:

- **Midterm 1**: Take-home. October 18–24; details TBA.
- **Midterm 2**: In-class, Wednesday, November 20.
- **Final**: Take-home. Details TBA.

The only excuses for missing an exam are incapacitating illness, religious conflict, or the like.

Using calculators, cell phones, etc. during an in-class exam is CHEATING.
Consulting any outside source during any exam is CHEATING.
Copying homework solutions, from any source, is CHEATING.
All cheating will be prosecuted, potentially leading to an F in the course.

Homework:

- Reading from the textbook will be assigned each week, along with short videos to watch.
- There will usually be two problem sets per week, each due **on gradescope by 11:59pm ET** on its due date, generally on Mondays and Thursdays.
See page 3 of this syllabus for important homework information.

Grading:

- **Effort**: 5%
- **Problem Sets**: 20%
- **Midterm Exams**: Better one: 25%. Worse one: 15%. (Total 40%.)
- **Final Exam**: 35%

"Effort" is a combination of class attendance (including being on time), class participation, and handing in problem sets. It is not computed linearly; a student deficient in any one of those areas will get a very low Effort grade. (See page 4 of this handout for more on attendance and participation.)

"Problem Sets" means actual grades on the problem sets. Late problem sets will be marked down substantially in the Problem Sets portion of your grade (see the webpage for details); but all problem sets handed in by the last day of classes count towards Effort.

If one component of an individual student's course grade is substantially higher or lower than his/her other grades, and if the student's Effort grade is strong, I will tweak the above percentages a little for that student to favor the better grades. Overall course grades will be curved.

About Cell Phones and Mobile Devices

Cell phones and similar devices have no place in my classroom. Don't use them. Not for talking, not for texting, not for anything. So at every class:

Silence your cell phone, put it away, and pay attention.

Necessary Background

The official prerequisite for this course is multivariable calculus (Math 211), which we will definitely use, especially partial derivatives and line integrals. But I would **strongly** recommend that you also have had some exposure to proofs. Here's a rough guide:

If you've taken	**your proof preparation for Math 345 is**
Math 350, 355	Excellent.
Math 225, 250, 255, or 280	Very good, especially if you've also had Math 220, 271, or 272.
Math 220 and 271, but **not** any other courses listed above	Good, if you got at least a B+ in both, or an A in at least one.
Math 220 and 272, but **not** any other courses listed above	Good, if you got at least an A- in both, or an A in at least one.
Math 220 **or** 271 **only**	Fine, if you got an A- or better.
Math 272 **only**	Fine if you got a solid A. Decent if you got an A-. But if you got B+ or lower, you should probably take something else first.
None of the courses above	Probably not enough. But feel free to talk to me about it.

If you aren't sure whether Math 345 would be appropriate for you, talk to me about it soon.

Course Content

Complex analysis is, loosely speaking, the study of calculus when the variables are complex numbers instead of real numbers. As in calculus with real numbers, we'll study limits, derivatives, integrals, sequences, and series. The complex numbers have some surprising properties: **geometrically**, the complex plane $\mathbb{C}$ looks like the familiar xy-plane $\mathbb{R}^2$; but **algebraically**, you can not only add and subtract but also *multiply and divide* in $\mathbb{C}$, unlike in the xy-plane $\mathbb{R}^2$. This combination leads to some amazing properties of differentiable functions from $\mathbb{C}$ to $\mathbb{C}$. Such functions are called **analytic functions**, and they will be the main focus of our study in this course.

Gamelin's three-part book covers everything from the definition of $\mathbb{C}$, through the basics and the not-so-basics of analytic functions, to a broad selection of some serious graduate-level applications of complex analysis. We'll get through most of the first part, as follows:

- Chapter I introduces $\mathbb{C}$, including both its algebraic and geometric properties. It continues with several examples of familiar functions like $f(x) = x^2$ and $g(x) = \sin x$, except now with x in $\mathbb{C}$. This chapter is heavy on computation and concepts and light on proofs.
- In Chapter II we'll introduce the property of analyticity and start proving things about analytic functions. Epsilons, limits, partial derivatives, and simple proofs will be everywhere.
- In Chapter III we'll talk about topics from multivariable calculus, like Green's Theorem. That's to get us ready for...
- ...Chapter IV, where we'll discuss complex line integrals and prove some very cool theorems.
- In Chapter V, we'll study power series, and especially power series expansions of analytic functions.
- In Chapter VI, we'll study Laurent series, which are a generalization of power series to functions with *poles*. For example, $x = 0$ is a pole of $f(x) = \cos(x)/x$ because f blows up near 0.
- In Chapter VII, we'll present the residue theorem, which allows us to compute certain kinds of integrals (including some integrals in *real* variables) more easily.

Homework

Your homework consists of **ALL THREE** of reading the book, watching the assigned videos, **AND** doing problem sets. Skim textbook sections **before** the corresponding lecture, and then go back and read more carefully to solidify your understanding and to help with the problem sets.

Please note the following **Important Problem Set Rules:**

1. Problem sets are due **on Gradescope**, each by its specified deadline.
2. You must "tag" your problems in Gradescope, marking where each problem's solution appears.
3. **Write legibly**, and organize your work clearly. **Make it a pleasure to read!**
4. If you worked with other students or got help from a source other than me, the book, or our Math Fellow (Karen), then say so explicitly on the first page of your problem set. (See the discussion below on the Statement of Intellectual Responsibility.)
5. The Problem Sets grade for any late assignment will be substantially reduced. The later it is, the greater the reduction; see the course webpage under "Problem Set Rules" for details.

See Homework 0 and the handout on Gradescope Instructions for more information on Gradescope.

I am often willing to grant penalty-free extensions on problem sets; but see "Attendance, Extensions, and Extra Office Hours" on page 4.

I strongly encourage you to work on problem sets together, in pairs or small groups, provided you follow the common-sense guidelines below.

About the Statement of Intellectual Responsibility

Exams: Your work must be entirely your own, so no looking at other people's papers, no communication, and no outside help. For the in-class midterm, no books, notes, calculators, or other resources are allowed, either. For the take-home exams, you may use only course materials, **your own** notes, and the textbook; you may also consult Prof. Benedetto, but not the TA or anyone else. No other books, notes, online resources, or communications with other people are allowed.

Problem sets: As I said above, I urge you to collaborate with each other, in pairs or small groups, under the following ground rules:

1. If you collaborate with, say, Jane and Joe, write a note on the front of your problem set saying, "I worked with Jane and Joe." Use similar notation if you got help from a tutor, fellow student, another professor, another book, the web, etc. However, you do **not** need to write about help you got from me, the Math 345 TA (Karen), the textbook, or course materials.
2. Working together does not mean that Joe does the first half of the problem set and Jane does the second half; everyone should work on every problem.
3. Each student must hand in their own problem set; you can't hand in a single packet as the work of multiple people. For Homework 0, by the way, "residue" is one of the secret words.
4. Each student must write up each problem **in their own words**. Working together means discussing the problems. Copying someone else's solution (even when the source doesn't mind) is plagiarism and a violation of intellectual responsibility.

A common question: What if Joe asks Jane about a homework problem she has already solved? If Joe copies Jane's solution, both Joe and Jane would be guilty of academic dishonesty, leading to an F in the course for both of them and potentially to dismissal from the college. Instead, Jane can explain her solution to Joe (even showing him what she wrote), before Joe writes up his own solution himself, **in his own words**. Joe would then have to write that he got help from Jane (see rule 1 above), but Jane doesn't need to write anything unless she also got help in return.

If at any time you aren't sure about what's OK and what's not as far as intellectual responsibility is concerned for this course, talk to me about it.

Class Attendance, Extensions, and Extra Office Hours

Attendance: You should be at every class meeting, and you should be on time. Of course, if you're sick, are in mandated isolation, have a religious conflict, or the like, just let me know (in advance, when possible). One or two accidental misses are OK, too; oversleeping can happen, but it should be **rare**. Otherwise, however,

I expect you to be in class, and on time, for every class meeting.

I don't plan to take formal attendance, but I will easily be able to tell who misses class too much; those students' Effort grades will take a hit. (And after more than just a very few unexcused absences, or showing up late too much, your Effort grade will truly plummet, since I do not compute it linearly.)

Extensions: You may request **two** homework extensions over the course of the semester, for up to 48 hours. To claim an extension, you must:

1. Not have used both your extensions yet,
2. Request the extension (by email, or during office hours or an appointment) **no later than 8pm ET on the due date**, and
3. Have been attending class devotedly and meeting homework deadlines.

Note: you do **not** need to provide an excuse or reason for your extension request.

Office Hours: you are always welcome to visit my regularly scheduled office hours. In addition, **IF you have been attending class and doing the homework**, you are also welcome to make appointments to see me **outside** of my regularly scheduled office hours.

Class Participation and Classroom Dynamics

Class participation is part of the Effort portion of your grade. If you are quiet by nature, don't worry; as long as you attend class devotedly, pay close attention, and do the homework, you will get full Effort credit. That said, finding your voice in class helps you learn better. So for your own benefit, **speak up, ask questions, and even try to answer questions I may ask in class**.

On the flip side,

respect your fellow students

both in and out of the classroom. Treat every person's ideas the same way you would if it had been me or the college president sharing them.

What to Expect

Math 345 will **not** be as hard as Math 350 or Math 355, but it will be noticeably **harder** than, say, Math 211, Math 220, or either of the linear algebra courses. We will definitely use some of the material from multivariable calculus, but topics from Math 121 will come up even more. In particular, sequences and series will figure prominently.

Complex analysis is, at first glance, very much like real analysis (i.e., calculus): there are limits, derivatives, integrals, sequences, and series. However, because of the kinds of phenomena that arise when you start to play with calculus over the complex numbers, the focus tends to be different. We'll be playing in a two dimensional space, which makes for some complications that simply don't arise in one-variable real analysis. On the other hand, complex analytic functions have such nice properties that other things become much easier. Some beautiful surprises are in store; stay tuned.