Math 345, Fall 2024 Professor Rob Benedetto

Review Sheet for Midterm Exam 2

Exam 2 will be **in class** on Wednesday, November 20. No books, notes, or other aids will be allowed. It will be designed for a 50-minute, closed book setting, rather than a week-long, open book setting. In particular, it will be shorter than Exam 1, with a larger proportion of computational problems, and the (very few) proof problems will be easier.

Briefly, you should know Sections IV.2–6, V.1–4, and V.6, along with a general knowledge of the stuff that came before.

The most important sections for this exam (since they have the most computational problems) are IV.4, V.2, V.3, V.4, and V.6.

The following is a more detailed list of most of the topics covered. I have included mentions of all chapters and sections since the start of the course, in part to emphasize what you *don't* need to know. However, please note that **THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.**

Old stuff to be familiar but which is not the focus of the exam

- Chapter I: Know the basics of complex numbers in Sections I.1 and I.2, and the definitions and rough ideas about the functions in Sections I.4–8. [But you **don't** need to know the stereographic projection, phase factors, or the Riemann surface stuff.]
- Chapter II: Be familiar with the definitions in Section II.1, and of course, know what analytic functions are (II.2). [But you **don't** need to know about the Cauchy-Riemann equations, harmonic functions, inverse functions, Jacobians, or conformal mappings.]
- Chapter III: You **don't** need to know anything specific from this chapter for this exam.
- Section IV.1: Know what a complex line integral is, and know the ML-estimate. [But you don't need anything else from this section.]
- IV.2: You **don't** need to know anything specific from this section for this exam.

Exam Content

- IV.3: Know Cauchy's Theorem.
- IV.4: Know the Cauchy Integral Formula and the Cauchy Differentiation Formula (page 114). Be able to compute integrals like those on page 116.
- IV.5: Know Liouville's Theorem. [But you **don't** need to know the "Cauchy Estimates" on the same page.]
- IV.6: Know Morera's Theorem. [But you **don't** need to know the other theorems in this section.]

(continued on next page)

Exam Content, continued

- V.1: Know what a series is, and what it means for one to converge. Know sums and scalar multiples of *convergent* series, absolute convergence, the comparison test, the divergence test, and the geometric series test. You should also know that the series $\sum 1/k^p$ converges if $p > 1$ and diverges if $p \leq 1$.

- V.2: Know the definitions of pointwise convergence and uniform convergence of a sequence or series of functions on a set E. Know that uniform convergence implies that we can switch limit and integral signs (or sum and integral signs), that the uniform limit of continuous functions is continuous, and that the uniform limit of analytic functions is analytic. Know the M-test. Know that if D is a domain, and if f_n are analytic and approach f uniformly on compact subsets of D (what the book calls "normal convergence" on D), then f is analytic, and the dervatives $f_n^{(m)}$ approach $f^{(m)}$ uniformly on compact subsets.

- V.3: Know what a power series is, and be able to find the radius of convergence of a given one. (See the examples on page 139, as well as the Ratio Test, Root Test, and Cauchy-Hadamard formula.) Know that a power series $\sum_{k \geq 0} a_k(z - z_0)^k$ is analytic on $D(z_0, R)$, where R is the radius of convergence. Be able to differentiate and anti-differentiate power series, and know that doing so does not change the radius of convergence.

- V.4: Know the Taylor series theorem (page 144), that an analytic function is equal to its Taylor series centered at z_0, at least on the disk $D(z_0, R)$, where R is the radius of convergence of the Taylor series. Know both formulas on page 144 (equations (4.2) and (4.3)) for computing the Taylor coefficients; note that (4.3) is really just the Cauchy Integral formula in disguise. Know the Taylor expansions of e^z, $\cos z$, $\sin z$, and $1/(1 - z)$ about $z_0 = 0$. Know the two Corollaries on page 146.

- V.6: Know how to add, subtract, multiply, and divide power series. (And differentiate and antidifferentiate, too, but we already mentioned that.)

Other possible things, and advice

- I may ask you to state one of the big theorems precisely: that would be **Cauchy**, **Cauchy Integral Formula**, **Morera**, **Liouville**, or the **M-Test**. To get full credit, you'd need to state all the hypotheses, etc.

- I won't ask you to re-prove any of the theorems from class or from the book; but it'd be good to be roughly familiar with the proofs, because, as you've probably noticed, some of the techniques are very useful in other contexts.

- Be able to recall and use theorems that aren't explicitly stated in the problem. For example, some line integrals are best done using the Cauchy Integral/Differentiation Formula, though I wouldn't tell you that in the statement of the problem. Or Liouville's theorem may be good for proving that some function is constant.