Math 345, Fall 2024 Professor Rob Benedetto

Homework #9

Due **Monday, October 7** in Gradescope by **11:59 pm ET**

- **WATCH** Video 10: Conformal Mappings
- **READ** Sections II.6 and II.7 of Gamelin
- **WRITE AND SUBMIT** solutions to the problems in this handout

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Problem 1. II.6, #5. Let $0 < B < \pi$. Find a conformal map of the wedge $\{-B < \operatorname{Arg} z < B\}$ onto the right half-plane $\{\operatorname{Re} w > 0\}$. (And of course, verify/prove all your claims.)

[*Note*: The point $z = 0$ does *not* belong to the wedge $\{-B < \operatorname{Arg} z < B\}$, since $\operatorname{Arg} 0$ is undefined.]

Problem 2. II.7, #1(c). Find the fractional linear transformation $f(z) = (az + b)/(cz + d)$ such that

$$f(\infty) = 0, \qquad f(1 + i) = 1, \qquad f(2) = \infty$$

Problem 3. II.7, #1(d). Find the fractional linear transformation $f(z) = (az + b)/(cz + d)$ such that

$$f(-2) = 1 - 2i, \qquad f(i) = 0, \qquad f(2) = 1 + 2i$$

Problem 4. II.7, #1(f). Find the fractional linear transformation $f(z) = (az + b)/(cz + d)$ such that

$$f(0) = 0, \qquad f(\infty) = 1, \qquad f(i) = \infty$$

Problem 5. II.7, #3. Let $f(z) = (az+b)/(cz+d)$ be the fractional linear transformation such that $f(1) = i$, $f(0) = 1 + i$, and $f(-1) = 1$.

Detemine the image of the following three sets under f:

- the unit circle $|z| = 1$.
- the open unit disk $|z| < 1$.
- the imaginary axis $\operatorname{Re}(z) = 0$.

Illustrate with a single sketch showing all three images. (And of course, as on every HW problem, (briefly) explain your reasoning.)

Optional Challenge:

II.7, #6: Let $L \subseteq \mathbb{C}$ be a straight line in the plane, and let $f(z) = 1/z$. Prove that the image of L under f is either a straight line (if L passes through the origin) or a circle (otherwise).

[*Note*: In this problem, consider a straight line to contain the point ∞.]