Math 345, Fall 2024 Professor Rob Benedetto

Homework #8

Due **Thursday, October 3** in Gradescope by **11:59 pm ET**

- **WATCH** Video 9: Derivatives of Inverse Functions
- **READ** Sections II.5 and II.6 of Gamelin
- **WRITE AND SUBMIT** solutions to the problems in this handout

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Problem 1. II.5, #1(a), variant. Prove that $u(x, y) = x^2 - y^2$ is harmonic on $\mathbb{C}$, and find a harmonic conjugate for it.

Find a simple formula (in terms of z) for an analytic function $f(z)$ for which $u = \operatorname{Re} f$.

Problem 2. II.5, #1(b), variant. Prove that $u(x, y) = xy + 3x^2y - y^3$ is harmonic on $\mathbb{C}$, and find a harmonic conjugate for it.

Find a simple formula (in terms of z) for an analytic function $f(z)$ for which $u = \operatorname{Re} f$.

Problem 3. II.5, #1(d), variant. Prove that $u(x, y) = e^{x^2-y^2}\cos(2xy)$ is harmonic on $\mathbb{C}$, and find a harmonic conjugate for it.

Find a simple formula (in terms of z) for an analytic function $f(z)$ for which $u = \operatorname{Re} f$.

Problem 4. II.5, #1(f), variant. Prove that $u(x, y) = x/(x^2 + y^2)$ is harmonic on $\mathbb{C} \setminus \{0\}$, and find a harmonic conjugate for it.

Find a simple formula (in terms of z) for an analytic function $f(z)$ for which $u = \operatorname{Re} f$.

Problem 5. II.6, #4. Let $A > 0$ be a positive real number. Find a conformal map of the horizontal strip $\{-A < \operatorname{Im} z < A\}$ onto the right half-plane $\{\operatorname{Re} w > 0\}$. (And of course, verify/prove all your claims.)

[*Hint*: Use the exponential function, and look back to some old homework problems where you looked at the images of certain regions under $w = e^z$.]

Optional Challenge:

II.5, #7: Prove that $\log |z|$ has no harmonic conjugate on the punctured plane $\mathbb{C} \setminus \{0\}$ (even though it *is* harmonic on that domain). Also prove that $\log |z|$ *does* have a harmonic conjugate on the slit plane $\mathbb{C} \setminus (-\infty, 0]$.