Math 345, Fall 2024 Professor Rob Benedetto

# Homework #20

Due **Tuesday, December 10** in Gradescope by **11:59 pm ET**

- **READ** Sections VII.2 and VII.4 of Gamelin
- **WRITE AND SUBMIT** solutions to the problems in this handout

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**Problem 1**. VII.2, #9. Show that $\displaystyle\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2+1}\,dx = \frac{\pi}{2}\left[1 - \frac{1}{e^2}\right]$.

**Problem 2**. VII.4, #1. Let $a \in \mathbb{R}$ with $0 < a < 1$. By integrating around the keyhole contour, show that

$$\int_0^{\infty} \frac{x^{-a}}{1+x}\,dx = \frac{\pi}{\sin(\pi a)}.$$

**Problem 3**. VII.4, #3. Let $a \in \mathbb{R}$ with $0 < a < 1$. By integrating around the keyhole contour, show that

$$\int_0^{\infty} \frac{\log x}{x^a(x+1)}\,dx = \frac{\pi^2\cos(\pi a)}{\sin^2(\pi a)}$$

[**Note**: It may come in handy at some point along the way to use the formula proven in the previous problem.]

**Problem 4**. VII.4 #3, continued, just for fun. Without worrying about switching orders of derivatives and integral signs, "check" the result of the previous problem by differentiating both sides of the formula in Problem 2 (i.e., VII.4 #1) with respect to $a$, to confirm that we get the formula in Problem 3.

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**Optional Challenge**: VII.4, #8. Integrate a branch of $(\log z)/(z^3+1)$ around the boundary of an indented sector of aperture $2\pi/3$, to show that

$$\int_0^{\infty} \frac{\log x}{x^3+1}\,dx = \frac{-2\pi^2}{27} \quad \text{and} \quad \int_0^{\infty} \frac{1}{x^3+1}\,dx = \frac{2\pi}{3\sqrt{3}}$$