Math 345, Fall 2024 Professor Rob Benedetto

# Homework #19

Due **Friday, December 6** in Gradescope by **11:59 pm ET**

- **WATCH** Video 24: The Casorati-Weierstrass Theorem
- **READ** Sections VII.1 and VII.2 of Gamelin
- **WRITE AND SUBMIT** solutions to the problems in this handout

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**Problem 1**. VII.1, #2(a). Calculate the residue of $f(z) = e^{1/z}$ at the isolated singularity at $z = 0$.

**Problem 2**. VII.1, #3(a,b). Use the Residue Theorem to evaluate the following integrals:

(a) $\displaystyle\oint_{|z|=1} \frac{\sin z}{z^2}\, dz$ (b) $\displaystyle\oint_{|z|=2} \frac{e^z}{z^2 - 1}\, dz$

**Problem 3**. VII.2 #2. Use residue theory to show that for any real constant $a > 0$, we have

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2 + a^2)^2} = \frac{\pi}{2a^3}.$$

**Problem 4**. VII.2 #7. Use residue theory to show that for any real constant $a > 0$, we have

$$\int_{-\infty}^{\infty} \frac{\cos(ax)}{x^4 + 1}\, dx = \frac{\pi}{\sqrt{2}} e^{-a/\sqrt{2}} \left( \cos\frac{a}{\sqrt{2}} + \sin\frac{a}{\sqrt{2}} \right).$$

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**Optional Challenge**: VII.2, #10. For $a, b \in \mathbb{R}$ with $b > 0$, show that

$$\int_{-\infty}^{\infty} \frac{\cos(ax)}{x^2 + b^2}\, dx = \frac{\pi e^{-|a|b}}{b}$$