Math 345, Fall 2024 Professor Rob Benedetto

Homework #18

Due **Tuesday, December 3** in Gradescope by **11:59 pm ET**

- **WATCH** Video 22: Laurent Expansion
- **WATCH** Video 23: Isolated Singularities
- **READ** Sections VI.4 and VII.1 of Gamelin
- **WRITE AND SUBMIT** solutions to the problems in this handout

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Problem 1. VI.4, #1(a,d). Find the partial fractions decompositions of the following functions.

(a) $\dfrac{1}{z^2 - z}$ (d) $\dfrac{1}{(z^2+1)^2}$

Problem 2. VI.4, #2(a). Use the division algorithm to (help) obtain the partial fractions decomposition of the function $\dfrac{z^3+1}{z^2+1}$

Problem 3. VII.1 #1(a,b,c). Evaluate the following residues.

(a) $\operatorname{Res}\left[\dfrac{1}{z^2+4}, 2i\right]$ (b) $\operatorname{Res}\left[\dfrac{1}{z^2+4}, -2i\right]$ (c) $\operatorname{Res}\left[\dfrac{1}{z^5-1}, 1\right]$

Problem 4. VII.1 #1(g,h). Evaluate the following residues.

(g) $\operatorname{Res}\left[\dfrac{z}{\operatorname{Log} z}, 1\right]$ (h) $\operatorname{Res}\left[\dfrac{e^z}{z^5}, 0\right]$

Optional Challenge: VI.4, #3. Let V be the complex vector space of functions that are analytic on the Riemann sphere $\overline{\mathbb{C}}$ except possibly at the points 0 and i, where they have poles of order at most 2. What is the dimension of V? Write down an explicit vector space basis of V.

[*Note*: A complex vector space is a vector space like $\mathbb{C}^n$ where the scalar multiplication is by elements of $\mathbb{C}$ rather than by elements of $\mathbb{R}$.]