Math 345, Fall 2025 Professor Rob Benedetto

Homework #3

Due **Monday, September 15** in Gradescope by **11:59 pm ET**

- **WATCH** Video 3: Some Riemann Surfaces (optional)
- **READ** Sections I.6–I.8 of Gamelin
- **WRITE AND SUBMIT** solutions to the following problems
  Don't forget that you must justify your claims.

Problem 1. (14 points) I.6, #1: Find and plot $\log z$ for each of the following $z \in \mathbb{C}$. Specify the principal value of each.

(a): $z = 2$ (a): $z = i$ (a): $z = 1 + i$ (d): $z = (1 + i\sqrt{3})/2$

Problem 2. (8 points) I.6, #2(a,b): Sketch the imagine under the map $w = \operatorname{Log} z$ of each of the following regions.

(a): the right half-plane $\operatorname{Re} z > 0$
(b): the half-disk $|z| < 1$, $\operatorname{Re} z > 0$

Problem 3. (5 points) I.6, #2(d): Sketch the imagine under the map $w = \operatorname{Log} z$ of the slit annulus $\sqrt{e} < |z| < e^2$, $z \notin (-e^2, -\sqrt{e})$

Problem 4. (10 points) I.7, #1(a,b): Find and plot all values of: (a) $(1+i)^i$. (b) $(-i)^{1+i}$.

Problem 5. (5 points) I.8, #1(a): Prove the identity $\cos(z+w) = \cos z \cos w - \sin z \sin w$.

Problem 6. (8 points) I.8, #4: Prove that $\tan^{-1} z = \dfrac{1}{2i} \log\left(\dfrac{1+iz}{1-iz}\right)$ by proving that $\tan w = z$ if and only if $2iw$ is one of the values of $\log\left(\dfrac{1+iz}{1-iz}\right)$.

Optional Challenges: I.7 #4,7

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Math Fellow Drop-in Hours (Katya Havryshchuk, SMUD 208):

Mondays	7:30–9:00pm
Wednesdays	7:30–9:00pm

Also, you may email me any time at `rlbenedetto@amherst.edu`