

Homework #20Due **Tuesday, December 9** in Gradescope by **11:59 pm ET**

- **READ** Sections VII.2 and VII.4 of Gamelin
 - **WRITE AND SUBMIT** solutions to the problems in this handout
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Problem 1. (20 points) VII.2, #9. Show that $\int_{-\infty}^{\infty} \frac{\sin^2 x}{x^2 + 1} dx = \frac{\pi}{2} \left[1 - \frac{1}{e^2} \right]$.

Problem 2. (25 points) VII.4, #1. Let $a \in \mathbb{R}$ with $0 < a < 1$. By integrating around the keyhole contour, show that

$$\int_0^{\infty} \frac{x^{-a}}{1+x} dx = \frac{\pi}{\sin(\pi a)}.$$

Problem 3. (30 points) VII.4, #3. Let $a \in \mathbb{R}$ with $0 < a < 1$. By integrating around the keyhole contour, show that

$$\int_0^{\infty} \frac{\log x}{x^a(x+1)} dx = \frac{\pi^2 \cos(\pi a)}{\sin^2(\pi a)}$$

[**Note:** It may come in handy at some point along the way to use the formula proven in the previous problem.]

Problem 4. (5 points) VII.4 #3, continued, just for fun. Without worrying about switching orders of derivatives and integral signs, “check” the result of the previous problem by differentiating both sides of the formula in Problem 2 (i.e., VII.4 #1) with respect to a , to confirm that we get the formula in Problem 3.

Optional Challenge: VII.4, #8. Integrate a branch of $(\log z)/(z^3 + 1)$ around the boundary of an indented sector of aperture $2\pi/3$, to show that

$$\int_0^{\infty} \frac{\log x}{x^3 + 1} dx = \frac{-2\pi^2}{27} \quad \text{and} \quad \int_0^{\infty} \frac{1}{x^3 + 1} dx = \frac{2\pi}{3\sqrt{3}}$$

(Office Hours on Next Page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays 2:00–3:30pm

Tuesdays 1:45–3:15pm

Fridays 1:00–2:00pm

or by appointment.

Math Fellow Drop-in Hours (Katya Havryshchuk, SMUD 208):

Mondays 7:30–9:00pm

Wednesdays 7:30–9:00pm

Also, you may email me any time at rlbenedetto@amherst.edu