Homework #2
Due **Thursday, September 11** in Gradescope by **11:59 pm ET**

- ☐ **WATCH** Video 2: Why $\mathbb{C}$ (Fund Thm of Alg) (on the moodle site)
- ☐ **READ** Sections I.3–I.5 of Gamelin
- ☐ **WRITE AND SUBMIT** solutions to the following problems
  Don't forget that you must justify your claims.

Problem 1. (7 points) I.4, #1(b): Sketch the curve $x = 1$ in the z-plane, and then sketch its image in the w-plane under $w = z^2$.

Problem 2. (7 points) I.4, #1(f): Sketch the curve $y = 1/x$ (for $x \neq 0$) in the z-plane, and then sketch its image in the w-plane under $w = z^2$.

Problem 3. (7 points) I.4, #2(b): Sketch the image of $x = 1$ under both branches of $w = \sqrt{z}$, in different colors.

Problem 4. (7 points) I.4, #3(a): Give a brief description of the function $w = z^3$, by describing what happens to a ray from the origin in the z-plane, and to a circle centered at the origin.

Problem 5. (7 points) I.4, #3(b) Make branch cuts and define explicitly three branches of the inverse mapping of $w = z^3$.

Problem 6. (6 points) I.5, #1(b,d): Calculate and plot e^z for the following points z:
$$\text{(b): } \pi i + 1 \qquad\qquad\qquad \text{(d): } 37\pi i$$

Problem 7. (4 points) I.5, #1(e): Calculate and plot e^z for $z = \pi i/m$ for $m = 1, 2, 3, \ldots$

Problem 8. (15 points) I.5, #2(a,b,c): Sketch each of the following figures (in the z-plane) and its image (in the w-plane) under $w = e^z$. Indicate the images (in the w-plane) of horizontal and vertical lines (from the z-plane).
  (a): the vertical strip $0 < \operatorname{Re} z < 1$
  (b): the horizontal strip $5\pi 3/ < \operatorname{Im} z < 8\pi/3$
  (c): the rectangle $0 < x < 1$, $0 < y < \pi/4$

Problem 9. (5 points) I.5, #3: Prove (for all $z \in \mathbb{C}$) that $e^z = \overline{e^{\bar{z}}}$

(Optional Challenges and Office Hour Information on next page)

Optional Challenges: I.3 #4,6; I.4 #4; and I.5 #4

Questions? You can ask in class or in:

My office hours (SMUD 406):
Mon, 2:00–3:30pm; Tue, 1:45–3:15pm; Fri, 1:00-2:00pm; or by appointment.

Also, you may email me any time at `rlbenedetto@amherst.edu`