Homework #19

Due **Friday, December 5** in Gradescope by **11:59 pm ET**

- **WATCH** Video 24: The Casorati-Weierstrass Theorem
- **READ** Sections VII.1 and VII.2 of Gamelin
- **WRITE AND SUBMIT** solutions to the problems in this handout

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Problem 1. (5 points) VII.1, #2(a). Calculate the residue of $f(z) = e^{1/z}$ at the isolated singularity at $z = 0$.

Problem 2. (12 points) VII.1, #3(a,b). Use the Residue Theorem to evaluate the following integrals:

(a) $\displaystyle\oint_{|z|=1} \frac{\sin z}{z^2}\, dz$ (b) $\displaystyle\oint_{|z|=2} \frac{e^z}{z^2-1}\, dz$

Problem 3. (20 points) VII.2 #2. Use residue theory to show that for any real constant $a > 0$, we have

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+a^2)^2} = \frac{\pi}{2a^3}.$$

Problem 4. (25 points) VII.2 #7. Use residue theory to show that for any real constant $a > 0$, we have

$$\int_{-\infty}^{\infty} \frac{\cos(ax)}{x^4+1}\, dx = \frac{\pi}{\sqrt{2}} e^{-a/\sqrt{2}} \left(\cos\frac{a}{\sqrt{2}} + \sin\frac{a}{\sqrt{2}} \right).$$

Optional Challenge: VII.2, #10. For $a, b \in \mathbb{R}$ with $b > 0$, show that

$$\int_{-\infty}^{\infty} \frac{\cos(ax)}{x^2+b^2}\, dx = \frac{\pi e^{-|a|b}}{b}$$

(Office Hours on Next Page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Math Fellow Drop-in Hours (Katya Havryshchuk, SMUD 208):

Mondays	7:30–9:00pm
Wednesdays	7:30–9:00pm

Also, you may email me any time at `rlbenedetto@amherst.edu`