Math 345, Fall 2025 Professor Rob Benedetto

# Homework #11

Due **Thursday, October 16** in Gradescope by **11:59 pm ET**

- **WATCH** Video 12: Closed and Exact
- (Optional) **READ** the handout on Path Independence on main course website
- **READ** Section IV.1 of Gamelin
- **WRITE AND SUBMIT** solutions to the problems in this handout

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**Problem 1**. (15 points) III.2, #5. Fix $0 < a < b$, and let $D = \{z \in \mathbb{C} : a < |z| < b\} \setminus (-b, -a)$, which is a slit annulus. Given points $A, B \in D$ and two paths $\gamma_0, \gamma_1$ from $A$ to $B$ in $D$, give an explicit homotopy [i.e., what Gamelin calls a family of paths $\gamma_s(t)$] from $\gamma_0$ to $\gamma_1$ in $D$.

[**Hint**: Use polar coordinates $re^{i\theta}$, not rectangular coordinates $x + iy$, to describe points in $D$.]

**Problem 2**. (18 points) IV.1, #1(a,b,c).
Let $\gamma$ be the boundary of the triangle $\{0 < x < 1,\ 0 < y < 1 - x\}$, oriented counterclockwise. Parametrize $\gamma$ and use your parametrization to compute each of the following integrals:

(a) $\displaystyle\int_\gamma \mathrm{Re}(z)\, dz$ (b) $\displaystyle\int_\gamma \mathrm{Im}(z)\, dz$ (c) $\displaystyle\int_\gamma z\, dz$

[**Suggestion**: Once you've computed the integrals in (a) and (b), part (c) is just a linear combination of them.]

**Problem 3**. (22 points) IV.1, #3, variant. Fix $R > 0$. Let $\gamma$ be the circle $\{|z| = R\}$, oriented counterclockwise. Parametrize $\gamma$ and use your parametrization to compute each of the following integrals for each integer $m = 0, \pm 1, \pm 2, \ldots$.

(a) $\displaystyle\int_\gamma z^m\, dz$ (b) $\displaystyle\int_\gamma |z|^m\, dz$ (c) $\displaystyle\int_\gamma \overline{z}^m\, dz$ (d) $\displaystyle\int_\gamma |z^m|\, |dz|$

[**Suggestion**: Once you've computed the integral in (a), you can use it to do the integrals in (b) and (c) fairly quickly.]

**Problem 4**. (15 points) IV.1, #5.
Use the $ML$-estimate to prove that $\displaystyle\left| \oint_{|z-1|=1} \frac{e^z}{z+1}\, dz \right| < 2\pi e^2.$

[**Note**: You need to prove $<$, not $\leq$. Incidentally, in reality, the value of this integral is 0, which is much smaller than $2\pi e^2$. But we won't be able to prove that easily until we learn Cauchy's Theorem in Section IV.3.]

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(Optional Challenge and Office Hour Information on next page)

**Optional Challenge**: IV.1, #9: Let $h(z)$ be a continuous function on a piecewise-smooth curve $\gamma$. Prove that the function

$$H(w) = \int_\gamma \frac{h(z)}{z-w}\,dz, \qquad w \in \mathbb{C} \smallsetminus \gamma$$

is analytic on $\mathbb{C} \smallsetminus \gamma$, and find $H'(w)$.

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**Questions?** You can ask in class or in:

**My (Drop-In) Office Hours** (SMUD 406):

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|---|---|---|
| ~~Mondays~~ | ~~2:00–3:30pm~~ | **Cancelled Monday, October 13** |
| ~~Tuesdays~~ | ~~1:45–3:15pm~~ | **Cancelled Tuesday, October 14** |
| Fridays | 1:00–2:00pm | |

or by appointment.

**This week only**: Wednesday, Oct 15 1:00–2:30pm

**Math Fellow Drop-in Hours** (Katya Havryshchuk, SMUD 208):

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|---|---|---|
| ~~Mondays~~ | ~~7:30–9:00pm~~ | **Cancelled Monday, October 13** |
| Wednesdays | 7:30–9:00pm | |

Also, you may email me any time at `rlbenedetto@amherst.edu`