Math 345, Fall 2025 Professor Rob Benedetto

Homework #1

Due **Monday, September 8** in Gradescope by **11:59 pm ET**

- **WATCH** Video 1: The Riemann Sphere (on the moodle site)
- **READ** Sections I.1 and I.2 of Gamelin
- **WRITE AND SUBMIT** solutions to the following problems
  Don't forget that you must justify your claims.

Problem 1. (7 points) I.1, #1(a,b): Identify and sketch the set of points $z \in \mathbb{C}$ satisfying:

(a): $|z - 1 - i| = 1$ (b): $1 < |2z - 6| < 2$

Problem 2. (8 points) I.1, #1(e,f): Identify and sketch the set of points $z \in \mathbb{C}$ satisfying:

(e): $|z - 1| < |z|$ (f): $0 < \operatorname{Im} z < \pi$

Problem 3. (7 points) I.1, #2(a,b): Verify the following identities from the definitions:

(a): $\overline{z + w} = \overline{z} + \overline{w}$ (b): $\overline{zw} = \overline{z}\,\overline{w}$

Problem 4. (4 points) I.1, #5, first half:
Prove (for all $z \in \mathbb{C}$) that $|\operatorname{Re} z| \leq |z|$ and $|\operatorname{Im} z| \leq |z|$.

Problem 5. (12 points) I.1, #5, second half:
Prove (for all $z, w \in \mathbb{C}$) that $|z + w|^2 = |z|^2 + |w|^2 + 2\operatorname{Re}(z\overline{w})$.
Then use this to prove the triangle inequality: $|z + w| \leq |z| + |w|$.

Problem 6. (8 points) I.2, #1(b,e): Express all values of the following expressions in both polar and cartesian coordinates, and plot them.

(b): $\sqrt{i - 1}$ (e): $(-8)^{1/3}$

[Note: Please evaluate trig functions at angles that are integer multiples of $\pi/6$ or $\pi/4$. However, leave other trig values, like $\sin(\pi/5)$ or $\cos(3\pi/8)$, as they are.]

Problem 7. (4 points) I.2, #2(a): Sketch the set of points $z \in \mathbb{C}$ for which $|\arg z| < \pi/4$.

Problem 8. (6 points) I.2, #4: For which integers $n \geq 1$ is i an n-th root of unity?
(Of course, prove that your answer is correct: that every integer $n \geq 1$ in the set you give has this property, **and** that no other $n \geq 1$ has this property.)

Problem 9. (4 points) I.2, #8, first part: Use DeMoivre's formulae to prove the standard trig identities $\cos(2\theta) = \cos^2\theta - \sin^2\theta$ and $\sin(2\theta) = 2\cos\theta\sin\theta$

Problem 10. (10 points) I.2, #8, second part: Find formulae for $\cos(4\theta)$ and $\sin(4\theta)$ (corresponding to those for 2θ in the previous problem), and again prove them using DeMoivre's formulae.

(Optional Challenges and Office Hour Information on next page)

Optional Challenges: I.1 #1(c), 8–11, and I.2 #5, 6

Questions? You can ask in class or in:

My office hours (SMUD 406):
Mon, 2:00–3:30pm; Tue, 1:45–3:15pm; Fri, 1:00-2:00pm; or by appointment.

Also, you may email me any time at `rlbenedetto@amherst.edu`