

Practice Problems for Midterm Exam 2

1. Compute the following integrals.

$$(a) \int_{|z|=6} \frac{\cos(\pi z)}{(z-1)^3} dz$$

$$(b) \int_{|z|=2} \frac{e^{i\pi z/2}}{z(z+1)(z+3)} dz$$

$$(c) \int_{|z-3|=2} \frac{\operatorname{Log} z}{z^2(z-4)^2} dz$$

$$(d) \int_{|z|=4} \frac{e^{5z}}{(z-\pi i)^3} dz$$

$$(e) \int_{|z|=\pi} \frac{e^{5z}}{(z-4)^4} dz$$

$$(f) \int_{|z-5|=4} \frac{(z-3)\sin z}{z^3(z-6)(z-8)} dz$$

2. For each of the following functions, find its full power series expansion about $z = 0$, as well as the radius of convergence of this power series.

$$(a) f(z) = z \operatorname{Log}(z+2)$$

$$(b) g(z) = \frac{z^2}{(z^5-4)^3}$$

3. Let $h(z) = (z^2 + 1)\sin(2z^3)$. Compute the following derivatives: $h^{(15)}(0)$, $h^{(16)}(0)$, and $h^{(17)}(0)$. [You do not need to simplify or expand expressions like $5^7 \cdot (17!)/(25!)$.]

4. (a) Find the power series centered at $z = 0$ of these two functions:

$$g(z) = ze^{(z^2)} \quad \text{and} \quad h(z) = \cos(2z).$$

(b) Consider the power series $\sum_{k=0}^{\infty} a_k z^k$ centered at $z = 0$ for $f(z) = \frac{ze^{(z^2)}}{\cos(2z)}$.

Use part (a) to compute a_k for each of $k = 0, \dots, 6$.

(c) For $f(z)$ as in part (b), compute $f^{(5)}(0)$.

(d) What is the radius of convergence of the power series in part (b)?
(Briefly) explain why.

5. Let $D = \{z \in \mathbb{C} : \operatorname{Re} z \leq -2\}$. Prove that $\sum_{n=1}^{\infty} n^z$ converges uniformly on D , where n^z denotes $e^{z \operatorname{Log} n}$.

6. Let $E = \{z \in \mathbb{C} : |z| \geq 7\}$. Prove that $\sum_{k=1}^{\infty} \frac{z^k}{5k - z^{3k}}$ converges uniformly on E .

7. Let f be an entire function with the property that for all $z \in \mathbb{C}$,

$$f(z+1) = f(z+i) = f(z).$$

Prove that f is constant.