Homework #8

Due **Wednesday, October 28** in Gradescope by **11:59 pm ET**

READ Textbook Sections 1.5.2–1.5.4, 1.6.1, and start 1.6.2

WATCH Optional Video 22: The Four Color Problem (19:52)

WRITE AND SUBMIT solutions to the following problems.

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1. (7 points) Textbook Section 1.5.2, #1:
Let G be a connected planar graph of order 24, and suppose that G is regular of degree 3. How many regions are there in a planar representation of G?

2. (15 points) Textbook Section 1.5.2, #4:
Let G be a connected planar graph of order $n \geq 3$, and suppose that G is K_3-free. (That is, G has no cycles of length 3.) Prove that the number q of edges of G satisfies $q \leq 2n - 4$.

3. (15 points) Textbook Section 1.5.2, #5:
Let G be a bipartite planar graph. Prove that $\delta(G) \leq 3$.
(*Suggestion*: suppose not, and use Problem 2, but remember that *this* problem doesn't require G to be connected.)

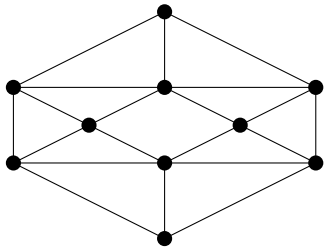
4. (14 points) Textbook Section 1.5.2, #7:
Let G be of order $n \geq 11$. Prove that at least one of G or $\overline{G}$ is nonplanar.

5. (15 points) Textbook Section 1.5.4, #2:
Use Kuratowski's Theorem to prove that the Petersen graph G is nonplanar. More specifically, show that G has a subgraph that is a subdivision of $K_{3,3}$.

6. (10 points) Textbook Section 1.6.1, #1(d):
Determine the chromatic number of the Petersen graph. As always, don't forget to justify your answer.

(problems continue on next page)

7. (10 points) Textbook Section 1.6.1, #1(e):
Determine the chromatic number of the Birkhoff Diamond, shown below. As always, don't forget to justify your answer.



8. (14 points) Textbook Section 1.6.1, #4 (slightly expanded):
Let G be a graph, let $e \in E(G)$, and let $G' = G - e$. Prove that $\chi(G') \leq \chi(G) \leq \chi(G') + 1$.

Optional Challenges (do NOT hand in): Textbook Section 1.5.2, Problem 10; Section 1.5.4, Problem 3

Questions? You can ask in:

Class: TuTh 8:35–9:50am

My office hours: Mon 1:45–3:15pm, Tue 1:45–3:15pm, and Fri, 1:00–2:00pm

Megan's Math Fellow office hours (SMUD 206):

Mondays, 7:30–9:00pm, and
Wednesdays, 7:30–9:00pm.

The QCenter also offers Graph Theory help:

Tim St. Onge (SCCE D111):

Mon–Fri, 10am–2pm
By appointment only (via Penji calendar).

Ris Paulino (SMUD 208):

Mon/Wed/Fri	10:00am–noon and 1:00pm–2:00pm
Tue/Thu	noon–3:00pm

or by appointment later in the afternoons

Also, you may email me any time at `rlbenedetto@amherst.edu`