Math 280, Fall 2026 Professor Rob Benedetto

Homework #3

Due **Wednesday, September 16** in Gradescope by **11:59 pm ET**

READ Textbook Sections 1.2.1 and 1.2.2

WATCH Video 11: Proving An Adjacency Corollary (8:33) [Found on moodle site]

Optional Video 12: Other Matrices Associated with a Graph (7:04)

WRITE AND SUBMIT solutions to the following problems.

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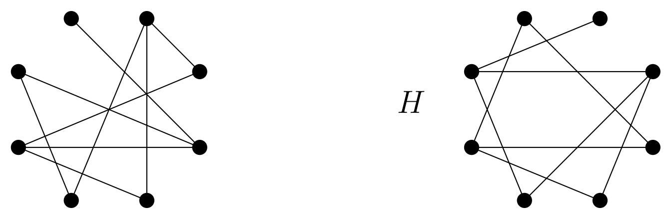
1. (10 points) Textbook, Section 1.1.3, Problem 9 (plus a little extra):
Consider the following two graphs:

G H



Verify that G and H have the same order, same size, and same degree sequence.
Then prove that in spite of that, G and H are *not* isomorphic.

2. (8 points) Suppose G is a graph that has 10 edges and 6 vertices, and suppose that the degrees of five of those vertices are $2, 2, 3, 4, 4$, and the sixth has some degree n.

(a) Find the integer n, i.e., the degree of the sixth vertex.

(b) Is G connected? (Yes, no, or maybe?) If "yes" or "no", prove it; if "maybe", draw two examples of such a graph G: one that is connected and one that is not.

3. (15 points) For each of the graphs P_5, C_5, and K_5:

(a) draw the graph

(b) find the eccentricity of each vertex

(c) find the radius and diameter of the graph

(d) find its adjacency matrix.

(For P_5, number the vertices 1 to 5 from one end to the other; for C_5, label them consecutively around the cycle.)

4. (10 points) Textbook, Section 1.2.1, Problem 5:
Let G be a graph, and let $u, v \in V(G)$ be adjacent vertices. Prove that their eccentricities $\mathrm{ecc}(u)$ and $\mathrm{ecc}(v)$ differ by at most 1.

(problems continue on next page)

5. (12 points) Textbook, Section 1.2.1, Problem 8(a,b,c):

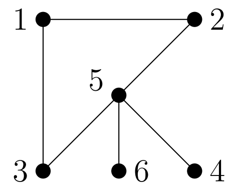
(a) Draw a graph of order 7 that has radius 3 and diameter 6.

(b) Draw a graph of order 7 that has radius 3 and diameter 5.

(c) Draw a graph of order 7 that has radius 3 and diameter 4.

In all three cases, don't forget to (briefly) justify that your graph has the correct order, radius, and diameter.

6. (15 points) Let G be the following graph:



(a) Find the adjacency matrix A of G.

(b) Find all the walks of length 3 from vertex 1 to vertex 4. What is the total number of such walks, and (without computing A^3) what does this say about the matrix A^3?

(c) How many closed walks of length 3 are there in G? Without computing A^3, how would this number be related to the matrix A^3?

(d) Find the eccentricities of all the vertices of G.

Optional Challenges (do NOT hand in):
Textbook Section 1.2.1, Problems 8(d), 10, 11; Section 1.2.2, Problems 4, 5

Questions? You can ask in:

Class: TuTh 8:35–9:50am

My office hours: Mon 1:45–3:15pm, Tue 1:45–3:15pm, and Fri, 1:00–2:00pm

Megan's Math Fellow office hours (SMUD 206):

Mondays, 7:30–9:00pm, and
Wednesdays, 7:30–9:00pm.

Tim St. Onge of the QCenter (SCCE D111) also offers Graph Theory help:

Mon–Fri, 10am–2pm
By appointment only (via Penji calendar).

Also, you may email me any time at `rlbenedetto@amherst.edu`