Homework #2

Due **Friday, September 11** in Gradescope by **11:59 pm ET**

READ $\boxed{\text{Textbook Section 1.1.3}}$ and start 1.2.1

WATCH $\boxed{\text{Video 6: Examples of Graphs}}$ (12:17) [Found on moodle site]

$\boxed{\text{Video 7: Two Sample Proofs}}$ (33:59)

$\boxed{\text{Video 8: Graph Isomorphisms}}$ (10:12)

$\boxed{\text{Optional Video 9: Distance on a Graph}}$ (15:25)

$\boxed{\text{Optional Video 10: Two Little Theorems}}$ (20:10)

WRITE AND SUBMIT solutions to the following problems.

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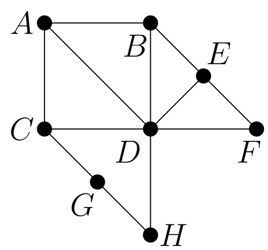
1. (12 points) Let X be the following graph:



(a) Find the degree sequence of X.
(b) Find a trail in X of longest possible length. Justify why there are no longer trails.
(c) Find a path in X of longest possible length. Justify why there are no longer paths.

2. (12 points) Textbook, Section 1.1.2, Problem 14:
Let G be a 2-connected graph. Prove that G contains at least one cycle.

(*Suggestion*: Remember that 2-connected means that if you delete any one vertex v, the subgraph $G-v$ is still connected, which means for any two of the remaining vertices, there's a path between them that avoids v.)

3. (10 points) Textbook, Section 1.1.2, Problem 16(a):
Let G be a graph of order n such that $\delta(G) \geq (n-1)/2$. Prove that G is connected.

4. (8 points) Textbook, Section 1.1.2, Problem 16(b):
For any positive even integer $n = 2m \geq 2$, find a graph G of order n such that $\delta(G) \geq (n-2)/2$ but G is *not* connected.

(*Suggestion*: For both Problems 3 and 4, think about what the two separate components of G would have to look like.)

(Problems continue on next page)

5. (12 points) Textbook, Section 1.1.3, Problem 3:
Is K_4 a subgraph of $K_{4,4}$? (More precisely, is there a subgraph of $K_{4,4}$ isomorphic to K_4, i.e., that is a complete graph on 4 vertices?) If yes, exhibit one explicitly. If no, prove no such subgraph exists.

6. (8 points) Textbook, Section 1.1.3, Problem 8:
Let G and H be isomorphic graphs. Prove that their complements $\overline{G}$ and $\overline{H}$ are also isomorphic.

Optional Challenges (do NOT hand in):
A. Textbook Section 1.1.3, Problem 10.
B. Let G be a graph with 10 vertices such that among any three vertices of G, at least two are adjacent. What is the minimum possible size (i.e., number of edges) that such a graph G can have? Find such a graph with this minimum number, and prove that no smaller number is possible.

Questions? You can ask in:

Class: TuTh 8:35–9:50am, SMUD 204

My office hours (SMUD 406):

Mon 1:45–3:15pm, Tue 1:45–3:15pm, and Fri, 1:00–2:00pm,
or by appointment.

Megan's Math Fellow office hours (SMUD 206):

Mondays, 7:30–9:00pm, and
Wednesdays, 7:30–9:00pm.

Tim St. Onge of the QCenter (SCCE D111) also offers Graph Theory help:

Mon–Fri, 10am–2pm
By appointment only (via Penji calendar).

Also, you may email me any time at `rlbenedetto@amherst.edu`