Math 280, Fall 2026 Professor Rob Benedetto

Homework #10

Due **Wednesday, November 18** in Gradescope by **11:59 pm ET**

READ Textbook Sections 1.7.1, 1.7.2, 1.7.4, and start 1.8.1.

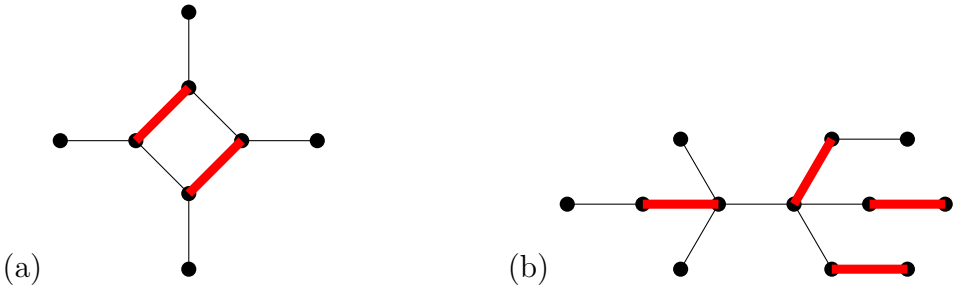
WRITE AND SUBMIT solutions to the following problems.

Problem 1. (18 points) Variant of Textbook Section 1.7.1, #2:

(a) (4 points) Find a perfect matching of C_{12}.

(b) (14 points) Find the minimum size of a maximal matching of C_{12}. That is, find a maximal matching M of C_{12} that has some number m of edges, and then prove that any *other* matching M' with $m-1$ or fewer edges cannot be maximal.

Problem 2. (10 points) Textbook Section 1.7.2, portions of #1(b):
For each of the following graphs, with matchings M as shaded, find an M-augmenting path, and use it to obtain a bigger matching.



(a) (b)

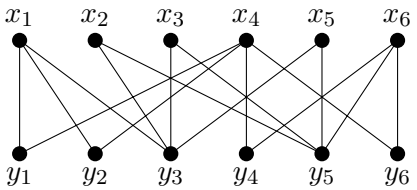
Problem 3. (15 points) Textbook Section 1.7.2, #2(a,b,d):
For each of the following families of sets, explicitly and carefully check whether the conditions of Theorem 1.52 are met. If so, then find an SDR, saying exactly which element is chosen from each set. If not, then show how the hypotheses are violated.

(a) $\{1,2,3\}$, $\{2,3,4\}$, $\{3,4,5\}$, $\{4,5\}$, $\{1,2,5\}$

(b) $\{1,2,4\}$, $\{2,4\}$, $\{2,3\}$, $\{1,2,3\}$

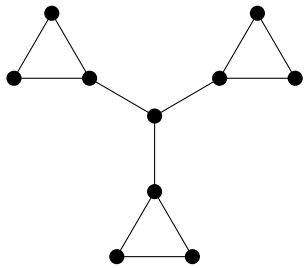
(d) $\{1,2,5\}$, $\{1,5\}$, $\{1,2\}$, $\{2,5\}$

Problem 4. (12 points) Use Hall's Theorem to prove that the following bipartite graph does not have a perfect matching.



(problems continue next page)

Problem 5. (14 points) Find a maximum matching of the following graph G, and prove that it is indeed a *maximum* matching.



Problem 6. (16 points) Textbook Section 1.7.4, #3:
Find and draw a connected, 3-regular graph that has both a cut vertex and a perfect matching. Don't forget to (briefly) verify that your graph has all these properties. (Connected, 3-regular, has a cut vertex, and has a perfect matching.)

Optional Challenges (do NOT hand in): Textbook Section 1.7.2, #5; Section 1.7.4, #1

Questions? You can ask in:

Class: TuTh 8:35–9:50am

My office hours: Mon 1:45–3:15pm, Tue 1:45–3:15pm, and Fri, 1:00–2:00pm

Megan's Math Fellow office hours (SMUD 206):

Mondays, 7:30–9:00pm, and
Wednesdays, 7:30–9:00pm.

The QCenter also offers Graph Theory help:

Tim St. Onge (SCCE D111):

Mon–Fri, 10am–2pm
By appointment only (via Penji calendar).

Ris Paulino (SMUD 208):

Mon/Wed/Fri	10:00am–noon and 1:00pm–2:00pm
Tue/Thu	noon–3:00pm

or by appointment later in the afternoons

Also, you may email me any time at `rlbenedetto@amherst.edu`