

Homework #1Due **Monday, September 7** in Gradescope by **11:59 pm ET****READ** Textbook Sections 1.1.1 and 1.1.2 and start 1.1.3**WATCH** Video 1: Sets and Motivating Examples (9:55) [Found on moodle site]Optional Video 2: Variants on Graphs (8:12)Video 3: Neighborhoods of Sets (7:47)Video 4: Subgraphs, Et Cetera (6:36)Video 5: Deleting Vertices or Edges (28:00)**WRITE AND SUBMIT** solutions to the following problems.

1. (8 points) Textbook, Section 1.1.2, Problem 1:

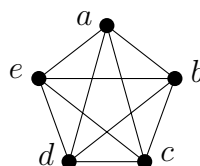
If G is a graph of order n , what is the maximum possible size of G ? (That is, what's the maximum possible number of edges G could have?)**Don't forget to justify your answer!** You don't need to give a formal proof on this more computational problem, but you need to explain why you know your answer is correct.

2. (12 points) Textbook, Section 1.1.2, Problem 2:

Let G be a graph of order $n \geq 2$. Prove that the degree sequence of G has at least one pair of repeated entries.*(Suggestion: What degrees are possible in such a graph G ? And use the pigeonhole principle.)*

3. (6 points) Textbook, Section 1.1.2, Problem 3(c):

Consider the following graph:



What is the maximum length of a circuit in this graph? Give an example of such a circuit.

Don't forget to justify your answer! You don't need to give a formal proof on this more computational problem, but you need to explain why you know your answer is correct.4. (7 points) Let G be a graph of odd order. Suppose that all the vertices of G have the same degree r . Prove that r is an even number.

5. (15 points) Textbook, Section 1.1.2, Problem 6:

Prove that every closed walk of odd length in a graph contains a cycle of odd length.

6. (7 points) Let G be a graph of order n and size t . Let $\overline{G}$ be the complement graph of G . (See Video 4, or item 3 of Section 1.1.3.) Find the order and size of $\overline{G}$.**Don't forget to justify your answers!**

(Optional Challenges and Office Hours on next page)

Optional Challenges (do NOT hand in): Textbook Section 1.1.2, Problems 5, 7, 8

Questions? You can ask in:

Class: TuTh 8:35–9:50am

My office hours: Mon 1:45–3:15pm, Tue 1:45–3:15pm, and Fri, 1:00–2:00pm

(Next week: Math Fellow office hours start, too!)

Also, you may email me any time at rlbenedetto@amherst.edu