Math 280, Fall 2026

Review Sheet for Midterm Exam 2

Briefly, you should know Sections 1.3.4, 1.4.2, 1.4.3, 1.5.1, and 1.5.2. More details are below. You still need to know general material covered on the first exam, of course, but the main focus of this second exam will be on the more recent topics from Section 1.3.4 to 1.5.2.
THIS IS NOT A COMPREHENSIVE LIST, BUT MERELY AN AID.

You may bring one standard size (8.5x11") "cheat sheet" of notes to the exam

You MAY use both sides of the sheet of paper for your cheat sheet.
You must HAND-WRITE your own cheat sheet directly on the paper.
No printouts.

- 1.3.4. Cayley's tree formula, and Prüfer's algorithms for associating labeled trees with Prüfer sequences (and vice versa). The Matrix Tree Theorem.
- 1.4.2. Eulerian trails and Eulerian circuits; a graph is *Eulerian* if it has an Eulerian circuit. Theorem 1.20, that a *connected* graph is Eulerian iff all its vertices have even degree. Corollary 1.21, with a similar iff condition for the existence of an Eulerian trail.
- 1.4.3. Hamiltonian paths and cycles. A graph is *traceable* if it has a Hamiltonian path, and *Hamiltonian* if it has a Hamiltonian circuit. Theorems 1.22, 1.24, 1.25. Independent set of vertices, and independence number $\alpha(G)$ of a graph G. For a specific graph H, what it means for another graph G to be H-free. The Petersen graph. The claw (i.e., $K_{1,3}$). For a graph G, its line graph $L(G)$. (Line graphs were originally defined in Exercise 7 of Section 1.1.3, page 16, but we didn't really discuss them until Section 1.4.3.)
- 1.5.1. Planar graphs. Regions of a planar graph. Bounding edges and non-bounding edges. The bound degree $b(R)$ of a region R.
- 1.5.2. Euler's formula for *connected* planar graphs. Theorems 1.33 and 1.35, giving inequalities involving planar graphs. The facts that $K_{3,3}$ and K_5 are nonplanar.

Some things you don't need to know

- 1.3.4: Proofs of Cayley's tree formula, of the Matrix Tree Theorem, and of the fact that Prüfer's algorithms work correctly.
- 1.4.1. You don't need to know anything in this section.
- 1.4.2. Proofs of the theorem and corollary on Eulerian trails and circuits. Hierholzer's algorithm.
- 1.4.3. Theorems 1.23 (Ore's Theorem) and 1.26 (of Duffus, Gould, and Jacobson). The specific graph N (in Figure 1.64). Proofs of all theorems in this section.
- 1.4.4. You don't need to know anything in this section.
- 1.5.1. Proofs of the theorems in this section.
- 1.5.2. Proofs of the theorems in this section, as well as the supplementary propositions I stated in class and videos on these topics.

Tips and other thoughts, same as from Exam 1

- To "know" the definition of a term means three things:
  - You have an intuitive, informal understanding of the concept.
  - You know the formal definition (not necessarily word for word, but close enough).
  - You can use the (formal) definition correctly in a rigorous proof.

  So make sure that applies to all of the (many) terms on the first page of this handout. In particular, be ready to use the definition of various terms in a short proof.

- I am **not** asking you to memorize any proofs of big theorems from class or from the book. But I **do** expect you to be familiar with the ideas, concepts, strategies, and methods that appear in many of those proofs.

- Yes, there will be proofs. But most of the problems on this timed, closed-book exam will be about analyzing examples. And the whole thing will be appropriate to a 65-minute, closed book exam; and you will have 75 minutes. So don't panic; just use the week to prepare.

- Besides reviewing your notes and the book, and familiarizing yourself again with the definitions and theorems, also do as many practice problems as you can: from old homework, and from the practice problems.

- Use office hours, both mine and Megan's.

- Spend the whole week studying, not just the last day or two, so that you can be sure to get a good night's sleep the night before the exam. Just an hour or two every day adds up, and doing so is much more effective than trying to cram for four hours the night before.