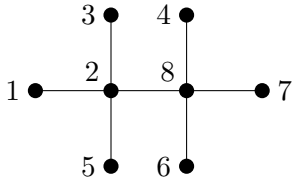
Math 280, Fall 2026

Practice Problems for Midterm Exam 2

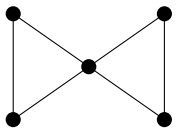
(A little more difficult, and much longer, than the real exam)

1. Use Prüfer's method to draw and label the trees with Prüfer sequences $1, 2, 3, 2, 1$ and $3, 2, 1, 2, 3$.

2. Use Prüfer's method to find the Prüfer sequence of this tree:



3. Use the Matrix Tree Theorem to compute the number of spanning trees of this graph:



4. Let G be the same graph as in problem 3 above.
   (a) Is G Eulerian? Prove your answer.
   (b) Is G Hamiltonian? Prove your answer.
   (c) What is the independence number of G? Prove your answer.
   (d) Is G claw-free? Prove your answer.
   (e) Is G C_3-free? Prove your answer.

5. Suppose G is Hamiltonian and $S \subseteq V(G)$ be a set of $m \geq 1$ vertices. Prove that $G - S$ has at most m connected components.

6. Suppose G is Hamiltonian. Prove that $\kappa(G) \geq 2$.

7. Give an example of a graph G of order at least 4 that is traceable (i.e., that has a Hamiltonian path) and has $\kappa(G) < 2$.

8. Prove that the graph $K_{4,4}$ is Eulerian but not planar.

9. Let G be the Petersen graph, and let $v \in V(G)$. (You may assume that v is the vertex at the top of the outer pentagon.) Prove that $G - v$ is Hamiltonian.

10. Let G be an Eulerian graph of order at least 2. Prove that the line graph $L(G)$ is Hamiltonian.

11. Let G be a 4-regular graph (i.e., each vertex has degree 4) that is connected and has order n. If G is planar, prove that the number of regions in a planar representation of G is $r = n + 2$.

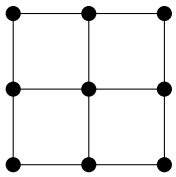
12. Let e_1 and e_2 be any two edges of K_6, and let $G = K_6 - e_1 - e_2$. Prove that G is not planar.

13. Let G be a Hamiltonian graph, let $m \geq 1$, and let $a, b, c, x_1, \ldots, x_m \in V(G)$ be $m + 3$ distinct vertices. Suppose that the neighborhood of a is

$$N(a) = \{b, c, x_1, \ldots, x_m\}$$

and that $\deg(b) = \deg(c) = 2$. Let $G' = G - e_1 - e_2 - \cdots - e_m$, where each e_i is the edge from a to x_i. Prove that G' is Hamiltonian.

14. Let G be the following graph:



Use the previous problem to prove that G is not Hamiltonian.

15. Let $r \geq 1$ be a positive integer, and G be a connected graph that is regular of degree r. [That is, all vertices of G have degree r.] Prove that the line graph $L(G)$ is Eulerian.

16. For any positive integers $a, b, c \geq 1$, the *complete tripartite graph* $K_{a,b,c}$ is the following graph. The vertex set is $A \cup B \cup C$, where A, B, C are pairwise disjoint sets of orders a, b, c, respectively. For any $a \in A$, $b \in B$, and $c \in C$, there are edges from a to b, from b to c, and from a to c; but there are no edges between any two vertices of A, any two vertices of B, or any two vertices of C.
Prove that $K_{a,b,c}$ is Eulerian if and only if a, b, c are either all even or all odd.

17. (Section 1.4.3, #6): Let G be a graph of order n that is r-regular for some $r \geq 1$. Prove that $\alpha(G) \leq n/2$.

18. (Section 1.5.2, #2): Let G be a connected planar graph of order at most 11. Prove that $\delta(G) \leq 4$.

19. Give an example of a graph G with an edge e such that G is claw-free but $G - e$ is *not* claw-free.

20. Give an example of a graph G with an edge e such that $G - e$ is claw-free but G is *not* claw-free.

21. Give an example of a graph G with a vertex v such that $G - v$ is claw-free but G is *not* claw-free.

22. Let G be a claw-free graph, and let $v \in V(G)$. Prove that $G - v$ is also claw-free.