Solutions to Homework #1

1. Section 1.1, Problem 1(a,b) (4 points)
   (a) True or False (and briefly explain why): $\{\text{Red}, \text{White}, \text{Blue}\} = \{\text{White}, \text{Blue}, \text{Red}\}$
   (b) Briefly explain what is wrong with this statement:
   Red is the first element of the set $\{\text{Red}, \text{White}, \text{Blue}\}$

Solutions. (a): True because both sets consist of exactly the same three elements.

(b): Elements of a set don't come in any particular order. That is, the order they are listed in here is arbitrary, so no particular element is the "first" one.

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2. Section 1.1, Problem 3(b–e) (4 points)
In each part, fill in the blank with the appropriate symbol, $\in$ or $\subseteq$:

(b) 3___$\{1, 2, 3, 4\}$ (c) $\{3\}$___$\{1, 2, 3, 4\}$

(d) $\{a\}$___$\{\{a\}, \{b\}, \{a, b\}\}$ (e) $\varnothing$___$\{\{a\}, \{b\}, \{a, b\}\}$

Solutions. (b): $\in$ (c): $\subseteq$ (d): $\in$ (e): $\subseteq$

FYI, to explain: For parts (b) and (d), the item on the left is one of the items listed in the set on the right. For parts (c) and (e), it is not, BUT in those parts, every element of the set on the left is also an element of the set on the right.
In particular, in part (e), when I say "every element of $\varnothing$", I mean "all none of them". It might feel weird to say that all the elements of the empty set are elements of the set on the right, but it's true, because every single one of them — that is, all none of them — is indeed in the other set.

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3. Section 1.1, Problem 6(a,b,d) (15 points)
Include brief justifications as you answer the following:

(a) (2 points): How many subsets does the empty set have?

(b) (3 points): How many subsets does the set $\{1\}$ have?

(d) (10 points): List all the subsets of the four-element set $\{1, 2, 3, 4\}$.

Solutions. (a): The empty set has one subset, namely $\varnothing$.

(b): The set $\{1\}$ has two subsets, namely $\varnothing$ and $\{1\}$.

(d): Here are the subsets of $\{1, 2, 3, 4\}$:

$\varnothing$, $\{1\}$, $\{2\}$, $\{3\}$, $\{4\}$, $\{1, 2\}$, $\{1, 3\}$, $\{1, 4\}$, $\{2, 3\}$, $\{2, 4\}$, $\{3, 4\}$, $\{1, 2, 3\}$, $\{1, 2, 4\}$, $\{1, 3, 4\}$, $\{2, 3, 4\}$, $\{1, 2, 3, 4\}$

Note on (d): that's a total of 16 subsets, which is 2^4. One fast way to see that there are 16 is to think about how to write down a subset U of $\{1, 2, 3, 4\}$: for each of the four elements 1,2,3,4, choose whether to put that element in U or not. So for each of the four possible elements, you have two choices: in or out. So with two choices for each of the four elements, that's a total of $2 \times 2 \times 2 \times 2 = 16$ ways to construct a subset $U \subseteq \{1, 2, 3, 4\}$.

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4. Section 1.1, Problem 10(a) (9 points) For each $k \in \{1, 2, \ldots, 20\}$,
let $D_k = \{x \mid x \text{ is a prime number that divides } k\}$.

Find the sets D_1, D_2, D_{10}, and D_{20}. (And give brief justifications.)

Solutions.
$D_1 = \varnothing$ (because no primes divide 1).

$\boxed{D_2 = \{2\}}$ because 2 is divisible by 2 but no other primes.

$\boxed{D_{10} = \{2, 5\}}$ because 10 is divisible by 2 and 5 but no other primes.

$\boxed{D_{20} = \{2, 5\}}$ because 20 is divisible by 2 and 5 but no other primes.

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5. Section 1.1, Problem 10(b(i–iii)) (8 points) With notation as in the previous problem: True or false (and briefly justify):

(i) $D_2 \subsetneq D_{10}$ (ii) $D_7 \subseteq D_{10}$ (iii) $D_{10} \subsetneq D_{20}$

Solutions.

(i): $\boxed{\text{True}}$ because $\{2\} \subsetneq \{2, 5\}$.

(ii): $\boxed{\text{False}}$ because $7 \in D_7$ (since 7 is prime and divides 7) but $7 \notin D_{10}$ (since 7 does not divide 10)

(iii): $\boxed{\text{False}}$ because $D_{10} = D_{20}$, since both sets are $\{2, 5\}$.