Homework #9
Due **Friday, October 10** in Gradescope by **11:59 pm ET**

READ | Sections 3.2, 3.3 in Richmond&Richmond |

WATCH 1. | Video 8: A GCD Theorem | (16:03)   [Found on moodle site]

2. | Video 9: The Euclidean Algorithm | (17:45)

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (8 points) Section 3.2, #4
Let $a, b, s, t, u, v \in \mathbb{Z}$ be integers such that $sa + tb = 21$ and $ua + vb = 10$.
Prove that $\gcd(a, b) = 1$.

Problem 2. (10 points) Section 3.2, #10
Let $a, b \in \mathbb{Z} \smallsetminus \{0\}$ be nonzero integers. Prove that
$\gcd(a, b) = 1$ if and only if $\gcd(a, a + b) = 1$.

Problem 3. (12 points) Section 3.3, #3
Prove Corollary 3.3.5(a): For any $m, n \in \mathbb{N}$, we have $mn = \gcd(m, n) \operatorname{lcm}(m, n)$

[You may, and should, use any of Theorem 3.3.2, Corollary 3.3.3, and Corollary 3.3.4.]

Problem 4. (10 points) Section 3.3, #9

Prove that for every $n \in \mathbb{N}$, we have $\dfrac{(3n)!}{3^n} \in \mathbb{N}$

Problem 5. (14 points) Section 3.3, #14
Let $a, d \in \mathbb{N}$. Prove that $d | a$ if and only if $d^2 | a^2$.

Problem 6. (16 points) Section 3.3, #15
Prove that for any $n \in \mathbb{N}$, we have that $\sqrt{n}$ either is an integer or is irrational.

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

      Mondays   2:00–3:30pm

      ~~Tuesdays~~   ~~1:45–3:15pm~~   **Cancelled Tuesday, October 7**

      Fridays   1:00–2:00pm

      or by appointment.

      This week only: Wednesday, Oct 8   1:00–2:30pm

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

      Mon/Wed/Fri   10:00am–noon

      Tue/Thu   1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

      Mondays   6:00–7:30pm   **Aaron** Cordoba

      Mondays   7:30–9:00pm   **John** Lim

      Tuesdays   6:00–7:30pm   **Aaron** Cordoba

      Tuesdays   7:30–9:00pm   **Gretta** Ineza

      Wednesdays   ~~7:30–9:00pm~~   **John** Lim

      This week only: Wednesday, Oct 8   9:00–10:30pm, SMUD 205

      Thursdays   6:00–7:30pm   **Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`