Homework #8

Due ~~Tuesday, September 30~~ **Wednesday, October 1** in Gradescope by **11:59 pm ET**

READ Sections 2.3 and 3.1 in Richmond&Richmond

WATCH 1. Video 6: Bad Induction (11:21) [Found on moodle site]

2. Video 7: The Strong Pigeonhole Principle (16:56)

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (12 points) Section 2.3, Problem 2
For this problem, you may give informal justifications (as opposed to formal proofs), but please mention where and how you are using the pigeonhole principle or related logic.
How many cards must be dealt from a standard deck of cards to guarantee:

(a) a pair, i.e., (at least) two cards of the same rank?

(b) a pair of aces?

(c) (at least) two cards of the same suit?

(d) (at least) five cards of the same suit?

[**Note**: a standard deck consists of 52 cards, including 13 from each of four suits (clubs, diamonds, hearts, and spades). The 13 cards in each suit come in different ranks (ace, 2 through 10, jack, queen, and king).]

Problem 2. (12 points) Section 2.3, Problem 3(a,b,c)
For this problem, you may give informal justifications (as opposed to formal proofs), but please mention where you and how are using the pigeonhole principle or related logic.
A piggy bank contains 12 pennies, 8 nickels, 10 dimes, and 3 quarters. How many coins must be grabbed from the bank to guarantee grabbing at least:

(a) 3 pennies?

(b) 3 coins of the same kind?

(c) 3 quarters?

Problem 3. (8 points) Section 2.3, Problem 7
For any set $S \subseteq \mathbb{Z}$ with $|S| = 3$, (i.e., for any set of three distinct integers), prove that S contains a pair whose sum is even (i.e., there exist distinct $m, n \in S$ such that $m + n$ is even).

Problem 4. (14 points) Section 2.3, Problem 9
Recall that the set $\mathbb{Z} \times \mathbb{Z}$ (of points in the plane both of whose coordinates are integers) is called the set of lattice points in the plane. For any five lattice points $(x_1, y_1), \ldots, (x_5, y_5) \in \mathbb{Z} \times \mathbb{Z}$, prove that there is a pair whose midpoint is also a lattice point.

[**Note**: The midpoint of two points (a, b) and (c, d) is the point exactly halfway along the line segment between the two points. The midpoint has coordinates $(\frac{a+c}{2}, \frac{b+d}{2})$.]

(problems continue on next page)

Problem 5. (10 points) Section 3.1, Problems 3(b) and 4

(a) [Problem 3b]: Let a, b, c, d be nonzero integers. Prove that if $a|b$ and $d|c$, then $(ad)|(bc)$

(b) [Problem 4]: Prove, or disprove via counterexample: Let a, b, c be nonzero integers. If $a|(bc)$, then $a|b$ or $a|c$.

Problem 6. (12 points) Section 3.1, Problem 12(a,b,e,h)
For each of the following pairs of integers a and b, find the integers $q, r \in \mathbb{Z}$ such that $b = qa + r$ and $0 \leq r < |a|$.

(a) $a = 73$, $b = 25$

(b) $a = 25$, $b = 73$

(e) $a = 79$, $b = -17$

(h) $a = 13$, $b = -37$

Problem 7. (10 points) Section 3.1, Problem 26
Prove that for every $n \in \mathbb{N}$, we have $6|(7^n - 1)$

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri	10:00am–noon
Tue/Thu	1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00–7:30pm	**Aaron** Cordoba
Mondays	7:30–9:00pm	**John** Lim
Tuesdays	6:00–7:30pm	**Aaron** Cordoba
Tuesdays	7:30–9:00pm	**Gretta** Ineza
Wednesdays	7:30–9:00pm	**John** Lim
Thursdays	6:00–7:30pm	**Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`