Math 220, Section 03, Fall 2025 Professor Rob Benedetto

Homework #7

Due **Friday, September 26** in Gradescope by **11:59 pm ET**

READ Sections 2.1, 2.2 in Richmond&Richmond

WATCH Video 5: Odds and Evens (19:05) [Found on moodle site]

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (6 points) Section 2.1, #22 (rephrased slightly)
For any rational number $x \in \mathbb{Q}$ and any irrational real number $y \in \mathbb{R} \smallsetminus \mathbb{Q}$, prove that $x + y$ is irrational.

Problem 2. (7 points) Section 2.1, #39, variant
We say an integer $z \in \mathbb{Z}$ is an *additive identity element for* $\mathbb{Z}$ (or simply an *additive identity*, for short) if the following statement is true:

$$\forall n \in \mathbb{Z}, \quad \text{we have} \quad z + n = n.$$

We saw a proof in class that 0 is an additive identity element for $\mathbb{Z}$. Prove that it is unique.

[That is, prove that if $z \in \mathbb{Z}$ is an additive identity element, then we must have $z = 0$.]

Problem 3. (20 points) Section 2.2, #2(b,h)
Prove the following by induction n.

(b) $1^3 + 2^3 + 3^3 + \cdots + n^3 = \dfrac{n^2(n+1)^2}{4}$ for all $n \in \mathbb{N}$

(h) $\dfrac{1}{0!} + \dfrac{1}{1!} + \dfrac{1}{2!} + \cdots + \dfrac{1}{n!} < 3 - \dfrac{2}{(n+1)!}$ for all $n \in \mathbb{N} \smallsetminus \{1\}$

Problem 4. (12 points) Section 2.2, #5
Find the value of $2^0 + 2^1 + 2^2 + \cdots + 2^n$ for $n = 0, 1, 2, 3, 4$. Make a conjecture about the value of the sum for $n \in \mathbb{N} \cup \{0\}$. Prove your conjecture.

Problem 5. (12 points) Section 2.2, #14
Prove *Bernoulli's inequality*: For every $\alpha \in \mathbb{R}$ with $\alpha > -1$ and $\alpha \neq 0$, and for every $n \in \mathbb{N} \smallsetminus \{1\}$, we have $(1 + \alpha)^n > 1 + n\alpha$.

Problem 6. (14 points) Section 2.2, #19 (with parts a, b, c)
Define a sequence of numbers $a_0, a_1, a_2, \ldots$ as follows:

$$a_0 = 0, \; a_1 = 1, \; \text{and for all } n \geq 2, \; \text{we have } a_n = 2a_{n-1} - a_{n-2} + 2.$$

(a) Find a_2, a_3, a_4, a_5

(b) Conjecture a formula for a_n for all $n \in \mathbb{N} \cup \{0\}$.

(c) Prove the conjectural formula you stated in part (b).

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri	10:00am–noon
Tue/Thu	1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00–7:30pm	**Aaron** Cordoba
Mondays	7:30–9:00pm	**John** Lim
Tuesdays	6:00–7:30pm	**Aaron** Cordoba
Tuesdays	7:30–9:00pm	**Gretta** Ineza
Wednesdays	7:30–9:00pm	**John** Lim
Thursdays	6:00–7:30pm	**Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`