Homework #6

Due **Tuesday, September 23** in Gradescope by **11:59 pm ET**

READ Section 2.1 in Richmond&Richmond

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (10 points) Section 2.1, #9
Let $a, b, c \in \mathbb{R}$, and let $p(x)$ be the polynomial $p(x) = ax^2 + bx + c$. Prove that $p(1) = p(-1)$ if and only if $p(2) = p(-2)$.

Problem 2. (10 points) Section 2.1, #12
Prove that for any integer $n \in \mathbb{Z}$, the number $n^3 + n$ is an even integer.

Problem 3. (10 points) Section 2.1, #15(a)
Let $a \in \mathbb{Z}$ be an integer. Prove that a is a multiple of 3 if and only if a may be written as the sum of three consecutive integers.

Problem 4. (15 points) Section 2.1, #17
Let $m, n \in \mathbb{Z}$ be integers. Prove that the following statements are equivalent:
(a) $m^2 - n^2$ is even.
(b) $m - n$ is even.
(c) $m^2 + n^2$ is even.

[**Note**: This is *not* a problem with three parts (a)–(c). Instead, you are being asked to prove that the three statements above are equivalent; that is, if any one of them is true, then all three are true.]

Problem 5. (15 points) Section 2.1, #5
Let $A, B, C \in \mathbb{Z}$ be integers with $A, B \neq 0$, and let L be the line $\{(x, y) \in \mathbb{R}^2 \mid Ax + By = C\}$. Suppose that L contains a lattice point [see below]. Prove that L contains infinitely many lattice points.

A *lattice point* in $\mathbb{R}^2$ is a point $(m, n) \in \mathbb{R}^2$ such that $m, n \in \mathbb{Z}$, i.e., such that both coordinates are integers.

Problem 6. (15 points) Section 2.1, #8(a)
For any $x \in \mathbb{R}$, let $\lfloor x \rfloor$ denote the greatest integer that is less than or equal to x. ($\lfloor x \rfloor$ is sometimes called the *floor function* of x.)
Prove that for all $x, y \in \mathbb{R}$, we have $\lfloor x \rfloor + \lfloor y \rfloor \leq \lfloor x + y \rfloor \leq \lfloor x \rfloor + \lfloor y \rfloor + 1$.

[**Hints/Suggestions for #6**: It may help to notice that for any $x \in \mathbb{R}$, if we define $m = \lfloor x \rfloor$, then $m \in \mathbb{Z}$ and $m \leq x < m + 1$. Also, if $k \in \mathbb{Z}$ is *some* integer with $k \leq x$, then we must have $k \leq m$, since m is the *greatest* integer with $m \leq x$.]

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri	10:00am–noon
Tue/Thu	1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00–7:30pm	**Aaron** Cordoba
Mondays	7:30–9:00pm	**John** Lim
Tuesdays	6:00–7:30pm	**Aaron** Cordoba
Tuesdays	7:30–9:00pm	**Gretta** Ineza
Wednesdays	7:30–9:00pm	**John** Lim
Thursdays	6:00–7:30pm	**Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`