Homework #5
Due **Friday, September 19** in Gradescope by **11:59 pm ET**

READ | Section 1.6 in Richmond&Richmond |

WATCH | Video 4: A Negated Quantifier Proof | (8:13)

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (12 points) Section 1.6, #2 (with parts a–c)
Consider the following statements:

I = Interest rates go down.
H = More people buy houses.
S = The stock market goes up.
U = Unemployment goes up.

Write each of the following implications, and its converse, and its contrapositive, in words:
  (a) $I \Rightarrow S$
  (b) $(\sim U) \Rightarrow H$
  (c) $S \Rightarrow (I \wedge H)$

Problem 2. (12 points) Section 1.6, #4(a–d)
Using the statements I, H, S, U above, write each of the following statements in symbolic form.
  (a) Interest rates go down only if unemployment goes up.
  (b) The stock market goes up if more people buy houses.
  (c) Unemployment does not go up if interest rates go down and more people buy houses.
  (d) If interest rates do not go down, the stock market does not go up.

Problem 3. (18 points) Section 1.6, #7(a,b,d)
Assume all variables in this problem represent real numbers. For each of the following implication statements, state its converse. In addition, determine the truth value of each implication and its converse, and briefly explain why.
  (a) $x^2 = 4$ only if $x = 2$.
  (b) If $2x \leq x$, then $x^2 > 0$.
  (d) If x is an integer, then $\sqrt{x}$ is an integer.

Problem 4. (18 points) Section 1.6, #8 (with parts a–c)
In all parts below, make sure to (briefly) justify your answer, as always.
  (a) Find a condition on $x \in \mathbb{R}$ which is necessary and sufficient for $x^2 \in [4, 9]$
  (b) Find a condition on $x \in \mathbb{R}$ which is necessary but not sufficient for $x^2 \in [4, 9]$.
  (c) Find a condition on $x \in \mathbb{R}$ which is sufficient but not necessary for $x^2 \in [4, 9]$.

Problem 5. (12 points) Section 1.6, #14(a,c)
Let A be a set, and suppose $R \subseteq A \times A$.

  (a) Find the contrapositive of the implication $\big[(a, b) \in R \wedge (b, a) \in R\big] \Rightarrow a = b$
  (c) Find the negation of the statement $\forall a, b \in A, \ \big[(a, b) \in R \wedge (b, a) \in R\big] \Rightarrow a = b$

Note: you do *not* need to prove or disprove any of these statements.

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri	10:00am–noon
Tue/Thu	1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00–7:30pm	**Aaron** Cordoba
Mondays	7:30–9:00pm	**John** Lim
Tuesdays	6:00–7:30pm	**Aaron** Cordoba
Tuesdays	7:30–9:00pm	**Gretta** Ineza
Wednesdays	7:30–9:00pm	**John** Lim
Thursdays	6:00–7:30pm	**Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`