Homework #4

Due **Tuesday, September 16** in Gradescope by **11:59 pm ET**

READ Sections 1.4, 1.5 in Richmond&Richmond

WRITE AND SUBMIT solutions to the following problems. **ALWAYS** justify your claims.

Problem 1. (5 points) Section 1.4, #7(a)
Verify the equation $\big(\sim(P \wedge Q)\big) \wedge (\sim Q) = \sim Q$ using a truth table.

Problem 2. (7 points) Section 1.4, #12, slight variant
Construct a truth table with columns for $(P \vee Q) \wedge R$ and $P \vee (Q \wedge R)$.
Briefly explain why it would be bad to use no parentheses when writing $P \vee Q \wedge R$.

Problem 3. (12 points) Section 1.5, #2(a,c,d)
Write each sentence below as a mathematical statement using $\forall$ and/or $\exists$.

(a) For every positive real number ε, there is a natural number n with $\frac{1}{n} < \varepsilon$
(c) For every positive real number ε, there is a positive real number δ such that $x^2 < \varepsilon$ whenever x is a real number with $|x| < \delta$.
(d) There exists an integer m with the property that for every integer x, there exists an integer y with $xy = m$.

Note: Please use mathematical symbols throughout your answers. For example, if the original sentence were "For every positive real number x, there is a negative real number y such that $y^2 = x$," then the mathematical statement would be:

$$\forall x \in \mathbb{R} \text{ s.t. } x > 0,\ \exists y \in \mathbb{R} \text{ with } y < 0 \quad \text{s.t. } y^2 = x.$$

Problem 4. (15 points) Section 1.5, #5(a,c,e)
For each statement below, determine whether it is true or false (and briefly explain why), and give the negation of the statement.

(a) $\forall x \in \mathbb{R}$, $\exists a \in \mathbb{R}$ with $|x| < a$
(c) $\forall x \in \mathbb{R}$, $\exists y \in \mathbb{R}$ s.t. $xy = 1$.
(e) $\forall a \in \mathbb{R}$, $\sqrt{a^2} = a$.

Problem 5. (15 points) Section 1.5, #6(a,c,d)
For each statement below, determine whether it is true or false (and briefly explain why), and give the negation of the statement.

(a) $\exists x \in \mathbb{Z}$ s.t. $\forall y \in \mathbb{Z}$, $\frac{y}{x} \in \mathbb{Z}$
(c) $\forall u \in \mathbb{N}$, $\exists v \in \mathbb{N} \smallsetminus \{u\}$ s.t. $\frac{v}{u} \in \mathbb{N}$
(d) $\forall u \in \mathbb{N}$, $\exists v \in \mathbb{N} \smallsetminus \{u\}$ s.t. $\frac{u}{v} \in \mathbb{N}$

Problem 6. (8 points) Section 1.5, #8(c)
For the following statement, determine whether it is true or false (and briefly explain why), and give the negation of the statement:

$$\forall k \in \mathbb{N},\ \exists S \in \mathcal{P}\big(\{1, 2, \ldots, k\}\big) \text{ s.t. } S \neq \varnothing \text{ and } \forall x, y \in S,\ x - y \text{ is even.}$$

(Office Hour Information on next page)

Questions? You can ask in class or in:

My (Drop-In) Office Hours (SMUD 406):

Mondays	2:00–3:30pm
Tuesdays	1:45–3:15pm
Fridays	1:00–2:00pm

or by appointment.

Allison Tanguay's QCenter Drop-in Hours (SMUD 208):

Mon/Wed/Fri	10:00am–noon
Tue/Thu	1:30–4:30pm

Math Fellow Drop-in Hours (SMUD 006):

Mondays	6:00–7:30pm	**Aaron** Cordoba
Mondays	7:30–9:00pm	**John** Lim
Tuesdays	6:00–7:30pm	**Aaron** Cordoba
Tuesdays	7:30–9:00pm	**Gretta** Ineza
Wednesdays	7:30–9:00pm	**John** Lim
Thursdays	6:00–7:30pm	**Gretta** Ineza

Also, you may email me any time at `rlbenedetto@amherst.edu`